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Many companies have embarked on initiatives that enable more demand information sharing between retailers and their upstream suppliers. While the literature on such initiatives in the business press is proliferating, it is not clear how one can quantify the benefits of these initiatives and how one can identify the drivers of the magnitudes of these benefits. Using analytical models, this paper aims at addressing these questions for a simple two-level supply chain with nonstationary end demands. Our analysis suggests that the value of demand information sharing can be quite high, especially when demands are significantly correlated over time.

(*Supply Chain Management; Mathematical Models; Production Planning and Inventory Control; Approximate Analysis; Electronic Data Interchange; Quick Response; Information Sharing*)

1. Introduction

Many industries have embarked on reengineering efforts to improve the efficiency of their supply chains. The goal of these programs is to better match supply with demand so as to reduce the costs of inventory and stockouts. The potential savings from such efforts can be astronomical, ranging from $14 billion for the food service industry (Troyer 1996) to $30 billion for the grocery industry (Kurt Salmon Associates 1993). One key initiative that is commonly mentioned is information sharing between partners in a supply chain. The literature on information sharing in the business press is proliferating. Nevertheless, although the benefits are intuitively clear, the literature is scant on the quantification of the benefits as well as the drivers of the magnitudes of these benefits. In this paper, we attempt to use a two-level supply chain model to show how the benefits can be quantified. In addition, we examine the underlying drivers that affect these magnitudes.

Sharing sales information has been viewed as a major strategy to counter the so-called "bullwhip effect" (see Lee et al. 1997a, 1997b).[1] The bullwhip effect is essentially the phenomenon of demand variability amplification along a supply chain, from the retailers, distributors, manufacturer, and the manufacturers' suppliers, and so on. Lee et al. characterize this phenomenon as *demand distortion*, which can create problems for suppliers, such as grossly inaccurate demand forecasts, low capacity utilization, excessive inventory, and poor customer service. By letting the supplier have visibility of point-of-sales data, the harmful effect of demand distortion can be amelio-

[1] Demand information sharing is often discussed in conjunction with Electronic Data Interchange (EDI). EDI is an enabler of demand information sharing, but demand information can also be transmitted through other communication means, such as by faxes (see the National Bicycles case as described in Fisher 1994). The increasing adoption of EDI has certainly helped the spread of demand information sharing (see Hammond 1993, EDI News 1995, Srinivasan et al. 1994, for the spread and benefits of EDI in the apparel industry, the grocery industry, and manufacturing sector, respectively).

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rated. The most celebrated implementation of demand information sharing is Wal-Mart's Retail Link program, which provides on-line summary of point-of-sales data to suppliers such as Johnson and Johnson, and Lever Brothers (Gill and Abend 1997). Indeed, demand information sharing by a downstream operator to his supplier is the cornerstone of initiatives such as Quick Response (QR) and Efficient Consumer Response (ECR). Often times, information sharing is embedded in programs like Vendor-Managed-Inventory (VMI) or Continuous Replenishment Programs (CRP). Major successes of such programs have been reported at companies like Campbell Soup (Clark 1994) and Barilla SpA (Hammond 1994).

The intent of this paper is two-fold. First, we develop a model of a two-stage supply chain that consists of a retailer and a manufacturer, and we analyze the benefit of information sharing to the chain. Our analyses suggest that information sharing alone could provide significant inventory reduction and cost savings to the manufacturer. With these savings, the retailer can negotiate arrangements with the manufacturer, such as the use of vendor managed inventory programs to reduce the retailer's overhead and processing costs, price reduction to reduce the retailer's variable cost, or lead time reduction to reduce the retailer's inventory cost, before sharing sales information. Second, our analyses and numerical examples suggest that the underlying demand process and the lead times have significant impact on the magnitudes of cost savings and inventory reductions associated with information sharing. Specifically, our results indicate that the manufacturer would experience great savings when: (a) the demand correlation over time is high; (b) the demand variance within each time period is high; or (c) the lead times are long. These conditions seem to fit the profile of most high-tech products. Therefore, our results suggest that information sharing would be especially useful for improving the efficiency of the supply chains in the high-tech industry.

There has been some recent interest in quantifying the value of information sharing between manufacturers and retailers (c.f., Bourland et al. 1996, Cachon and Fisher 1997, Gavirneni et al. 1999). First, Bourland et al. examine the case in which the review period of the manufacturer is not synchronized with the retailer. Because of this difference in review periods, the manufacturer can determine the order replenishment decision by making use of the inventory level at the retailer at the time of its order review. Similarly, Cachon and Fisher show analytically how the manufacturer can benefit from using information about the retailer's inventory levels when the retailers use a batch ordering policy. These two models capture the value of partial uncertainty resolution by gaining demand information at the retailers. Next, Gavirneni et al. consider the case in which the manufacturer has limited capacity. In addition, they consider two cases of information sharing between the manufacturer and the retailer. In the first case, the manufacturer obtains information from the retailer about the parameters of the underlying demand distribution and the value of the (s, S) ordering policy adopted by the retailer. In the second case, the manufacturer obtains additional information from the retailer about the period-to-period inventory level. Gavirneni et al. compare the cost between the first and the second case so as to evaluate the benefit of obtaining additional information about the retailer's inventory level. By considering various types of demand distributions in their numerical experiments, they examine the conditions under which gaining information about the retailer's inventory level is beneficial. All of these three research articles are based on demand processes that are independent and identically distributed over time. Thus, the benefit of information sharing lies in the manufacturer's capability to react to the retailer's needs via the knowledge of the retailer's inventory levels to help reduce uncertainties in the demand process faced by the manufacturer. Relative to the existing literature that examines the benefit of information sharing, our current paper examines a different situation in which the underlying demand process is autocorrelated. When the underlying demand process is autocorrelated, the manufacturer can benefit from obtaining information about the demand from the retailer because it would enable the manufacturer to derive a more accurate forecast of future orders placed by the retailer. In this paper, we examine the impact of the autocorrelation coefficient and the leadtime on the benefit of information sharing in a two-stage supply chain.

This paper is organized as follows. The next section presents a model of a two-level supply chain. By analyzing the retailer's and the manufacturer's ordering decisions (with information sharing and without information sharing), we develop expressions for the optimal order-up-to level for the retailer and the manufacturer in §3. In §4, we first analyze the benefits (cost savings and inventory reduction) of information sharing, and determine the underlying factors that have significant impact on the benefits of information sharing. Then we present some numerical examples to illustrate the benefits of information sharing under different scenarios. Section 5 examines the impact of the demand process on the benefits of information sharing. In §6, we study the impact of lead time reduction on the benefit of information sharing. The paper ends with a discussion.

2. The Modeling Framework

Consider a simple two-level supply chain that consists of one manufacturer and one retailer.[2] External demand for a single item occurs at the retailer, where the underlying demand process faced by the retailer is a simple autocorrelated AR(1) process.[3] Let D_t be the AR(1) demand process at the retailer, where

$$D_t = d + \rho D_{t-1} + \epsilon_t, \tag{2.1}$$

$d > 0$, $-1 < \rho < 1$, and ϵ_t is *i.i.d.* normally distributed with mean zero and variance σ^2. We further assume that σ is significantly smaller than d, so that the probability of a negative demand is negligible. Note that $\rho = 0$ corresponds to the special case in which the demand for each time period is *i.i.d.*

We consider a periodic review system in which each site reviews its inventory level and replenishes its inventory from the upstream site every period.[4] We assume that the replenishment leadtimes from the external supplier to the manufacturer, and from the manufacturer to the retailer, are in constant periods and denoted by L and l, respectively. (Throughout this paper, parameters represented in upper case and lower case are designated for the manufacturer and the retailer, respectively.)

First, let us describe the retailer's ordering process. Before the end of time period t, $t = 1, 2, 3, \ldots$, after demand D_t has been realized, the retailer observes the inventory level and places an order of size Y_t with the manufacturer to replenish his inventory. The retailer will receive the shipment of this order at the beginning of time period $t + l + 1$. Excess demand is backlogged.

Next, the manufacturer handles his ordering process as follows. At the end of time period t, the manufacturer receives and ships the required order quantity Y_t to the retailer. If the manufacturer does not have enough stock to fill this order, then we assume that the manufacturer will meet the shortfall by obtaining some units from an "alternative" source, with additional cost representing the penalty cost to this shortfall.[5] We consider the case in which the manufacturer is solely responsible for the penalty cost and for *resupplying* this alternative source later.[6] Thus,

[2] Our model can be extended to the case in which there is one manufacturer and multiple retailers.

[3] The approach presented in this paper can be extended to analyze more general demand processes such as AR(n) process; however, the analysis would become complex. Since our intent is to obtain some basic managerial insights, we shall restrict our attention to the AR(1) process only. Inventory models that assumed AR(1) demand process include Kahn (1987) and Miller (1986).

[4] The model implications remain the same when the retailer and the manufacturer review their inventory systems every 2 or 3 periods. However, for the general periodic review system, the analysis becomes quite intractable. The reader is referred to Lee et al. (1996) for details.

[5] This assumption can also be viewed as an approximation of a system with no alternative source. Gallego and Zipkin (1998) show how this assumption enables us to decompose a multiple-stage system with no alternative source into single-stage systems. Their numerical analysis implies that this assumption enables us to approximate the cost of the system with no alternative source within 10%.

[6] In reality, the shortage faced by the manufacturer could result in delays of supply to the retailer. Alternatively, the cost of expediting the shortage from the manufacturer to the retailer may be shared by both the manufacturer and the retailer. In the current paper, we make the assumption that the expedite cost is borne solely by the manufacturer so as to isolate the benefits of information sharing to the manufacturer. A similar assumption was made by most other researchers, such as Gavirneni et al. (1999) and Bourland et al. (1996). If this assumption is relaxed, then information sharing could

the inventory system at the manufacturer resembles a system with back orders, and the manufacturer guarantees supply to the retailer.

We assume that no fixed order cost is incurred when placing an order, and that unit inventory holding cost and shortage cost are stationary over time. Let h and p denote the unit holding and shortage costs per time period for the retailer, respectively. Let H and P denote the unit holding and shortage (or back order) costs per time period at the manufacturer, respectively. The shortage cost at the manufacturer represents the penalty cost to the manufacturer for obtaining items from the alternative source. We assume that the retailer and the manufacturer would also adopt the order-up-to policy, since such a policy minimizes the total discounted holding and shortage costs over the infinite horizon (see Heyman and Sobel (1984), Kahn (1987), and the discussions in Lee et al. (1997a)).

3. Ordering Decisions

Our approach for evaluating the benefit of information sharing is as follows. For any given AR(1) demand process, we first analyze the retailer's order quantity in this section. Then, by treating the retailer's order quantity as the *demand process* for the manufacturer, we analyze the manufacturer's order quantity for two cases (no information sharing, and with information sharing). In §4, we compare the manufacturer's order quantity for these two cases and evaluate the inventory reduction and cost savings associated with information sharing.

We now develop the expressions for the optimal ordering decisions for the retailer and the manufacturer. These expressions would allow us to examine the benefits of information sharing in §4.

bring benefits to both the manufacturer and the retailer, but this requires much more complex modeling of the contractual relationship between the manufacturer and the retailer. For example, one would have to address the issue of why a retailer should bear the cost of a nonperforming manufacturer (in terms of on-time delivery), and the differential negotiation power of the two partners. Hence, following Gavirneni et al. (1999) and Bourland et al. (1996), we assume that the expedite cost is borne solely by the manufacturer and as a result, we were able to concretely derive the value of information sharing.

3.1. Retailer's Ordering Decision

Consider first the retailer's ordering decision. Let S_t, $t = 1, 2, 3, \cdots$ denote the retailer order-up-to level. At the end of time period t, the retailer orders Y_t, where:

$$Y_t = D_t + (S_t - S_{t-1}). \tag{3.1}$$

In other words, the order quantity Y_t replenishes the demand during period t plus the change being made in the order-up-to levels. Notice from (3.1) that it is possible to have $Y_t < 0$. However, under the assumption that σ is significantly smaller than d, Lee et al. (1997a) show that the probability of having $Y_t < 0$ is negligible. Notice that the assumption that $Y_t > 0$ is automatically satisfied if returns are allowed without additional costs. Moreover, as we shall show later, the assumption that $Y_t \geq 0$ is less stringent than the conventional assumption being made in the traditional periodic review inventory system with normally distributed demand.

We now derive the expression for the order-up-to level S_t that minimizes the total expected holding and shortage costs in period $t + l + 1$. First, by using the recursive relationship of D_t given in (2.1), the total demand over the lead time, denoted by $\sum_{i=1}^{l+1} D_{t+i}$, can be expressed as:

$$\begin{aligned}\sum_{i=1}^{l+1} D_{t+i} = \frac{1}{1-\rho}\{d \sum_{i=1}^{l+1} (1-\rho^i) + \rho(1-\rho^{l+1})D_t\} \\ + \epsilon_{t+l+1} + (1+\rho)\epsilon_{t+l} + \cdots \\ + (1+\rho+\rho^2+\cdots+\rho^l)\epsilon_{t+1}.\end{aligned}$$

Let m_t and v_t be the conditional expectation and the conditional variance of the total demand over the lead time, respectively, where $m_t = E(\sum_{i=1}^{l+1} D_{t+i}|D_t)$ and $v_t = \mathrm{Var}(\sum_{i=1}^{l+1} D_{t+i}|D_t)$. It can be shown that:

$$m_t = \frac{d}{1-\rho}\{(l+1) - \sum_{j=1}^{l+1} \rho^j\} + \frac{\rho(1-\rho^{l+1})}{1-\rho} D_t, \tag{3.2}$$

$$v_t = v\sigma^2, \tag{3.3}$$

where

$$v = \sum_{j=1}^{l+1} \left\{ \sum_{i=0}^{j-1} \rho^i \right\}^2 = \frac{1}{(1-\rho)^2} \sum_{j=1}^{l+1} (1-\rho^j)^2. \tag{3.4}$$

In this case, the retailer's order-up-to level S_t is given as:

$$S_t = m_t + k\sigma\sqrt{v},$$

where $k = \Phi^{-1}[p/(p + h)]$ for the standard normal distribution function Φ. From (3.1) and the above expression for S_t, the retailer's order quantity Y_t can be written as:

$$Y_t = D_t + \frac{\rho(1 - \rho^{l+1})}{1 - \rho}(D_t - D_{t-1}). \qquad (3.5)$$

Let us consider the special case in which $\rho = 0$. It can be easily seen from (3.2), (3.4), and (3.5) that $m_t = (l + 1)d$, $v_t = (l + 1)\sigma^2$, $S_t = (l + 1)d + k\sigma\sqrt{l + 1}$, and $Y_t = D_t$. Thus, when $\rho = 0$, the retailer's order-up-to order quantity S_t is a constant that does not depend on the actual realization of the demand. In addition, since the retailer orders every period, the retailer would order according to the demand realized in period t. As such, the "bull whip" effect does not occur when the autocorrelation coefficient $\rho = 0$.

We now argue that our assumption $Y_t \geq 0$ is less stringent under an AR(1) demand process than under the traditional periodic review inventory system with *i.i.d.* normally distributed demand. To do so, we shall show that, as $t \to \infty$, the probability of $(Y_t \geq 0)$ is increasing in ρ for $\rho \geq 0$ such that the probability of having our assumption being valid (i.e., $Y_t \geq 0$ with $\rho > 0$) is higher than that under the traditional model with *i.i.d.* demands (i.e., $\rho = 0$). This result is captured in the following lemma.

LEMMA 1. *For $\rho \geq 0$, $P\{Y_t \geq 0\}$ is increasing in ρ as $t \to \infty$.*

The above lemma implies that our assumption $Y_t \geq 0$ is even less stringent as ρ increases.

3.2. Manufacturer's Ordering Decision

Now consider the manufacturer's ordering decision. We assume that the manufacturer is aware of the fact that the demand process D_t follows an AR(1) process, with known parameter d, ρ, and σ. This assumption is reasonable, as such information about the underlying demand process can be communicated to the manufacturer through periodic discussion with the retailer, or the manufacturer can be provided with historic demand data from which such information can be readily deduced with sufficient accuracy. As shown in Gavirneni et al. (1999), the cost savings would be even higher under information sharing when the manufacturer has no or partial knowledge about the underlying process.

When the manufacturer knows the parameters associated with the underlying AR(1) process D_t, the manufacturer can utilize (3.5) to estimate the actual value of D_t. Consequently, the value of obtaining information about the actual demand from the retailer will be reduced. Since it is complex to analyze the value of information sharing analytically for the case when the manufacturer utilizes historical order quantities to estimate the actual demand, we shall limit the scope of our paper by assuming that the manufacturer would not utilize (3.5) to infer the actual value of D_t.

After the manufacturer receives and ships the retailer's order Y_t at the end of time period t, the manufacturer immediately places an order with his supplier at the end of time period t so as to bring his inventory position to an order-up-to level T_t. This order will arrive at the beginning of period $t + L + 1$ to be ready for the retailer's order placed at the end of period $t + L + 1$.

To determine his order-up-to level T_t, the manufacturer needs to anticipate his total "demand" (or shipment quantity) over the manufacturer's lead time. Since the manufacturer's "demand" corresponds to the retailer's order quantity, the total shipment quantity over the manufacturer's lead time, denoted by $\mathcal{B}_t$, is equal to the total orders placed by the retailer over the time period $t + 1, \ldots, t + L + 1$. Specifically, $\mathcal{B}_t = \sum_{i=1}^{L+1} Y_{t+i}$. To determine the conditional mean and the conditional variance of $\mathcal{B}_t$ given the retailer's order Y_t, let us develop an expression of $\mathcal{B}_t$ in terms of Y_t. To do so, observe from (2.1) and (3.5) that the manufacturer can deduce that:

$$Y_{t+1} = d + \rho Y_t + \frac{1 - \rho^{l+2}}{1 - \rho}\epsilon_{t+1} - \frac{\rho(1 - \rho^{l+1})}{1 - \rho}\epsilon_t. \qquad (3.6)$$

Repeated use of (3.6) yields:

$$Y_{t+i} = \frac{1 - \rho^i}{1 - \rho} d + \rho^i Y_t + \frac{1 - \rho^{l+2}}{1 - \rho}\epsilon_{t+i} + \sum_{k=1}^{k=i-1} \rho^{l+1+k}\epsilon_{t+i-k} - \frac{\rho^i(1 - \rho^{l+1})}{1 - \rho}\epsilon_t,$$

$$i = 1, 2, \ldots,$$

where $\Sigma_a^b = 0$ if $a > b$. By using the above equation and simplifying, we can express $\mathcal{B}_t$, the total shipment quantity over the manufacturer's lead time for any given value of Y_t, as:

$$\begin{aligned}\mathcal{B}_t &= \sum_{i=1}^{L+1} Y_{t+i} \\ &= \frac{d}{1-\rho}\left\{(L+1) - \frac{\rho(1-\rho^{L+1})}{1-\rho}\right\} \\ &\quad + \frac{\rho(1-\rho^{L+1})}{1-\rho} Y_t + \frac{1-\rho^{l+2}}{1-\rho}\epsilon_{t+L+1} \\ &\quad + \frac{1}{1-\rho}\sum_{i=1}^{L}(1-\rho^{L+l+3-i})\epsilon_{t+i} \\ &\quad - \frac{\rho(1-\rho^{L+1})(1-\rho^{l+1})}{(1-\rho)^2}\epsilon_t. \end{aligned} \tag{3.7}$$

In order to determine the manufacturer's order-up-to level T_t that minimizes the total expected inventory holding and shortage costs in period $t + L + 1$, the manufacturer needs to find the distribution of $\mathcal{B}_t$.

3.2.1. No Information Sharing. When there is no information sharing, the manufacturer receives only information about the retailer's order quantity Y_t. In this case, the error term ϵ_t has already been realized, but is unknown to the manufacturer when he determines his order-up-to level T_t at the end of period t. Thus, it follows from (3.7) that the manufacturer would treat $\mathcal{B}_t$, the manufacturer's total shipment quantity over the manufacturer's lead time, as having a normal distribution F_t with mean M_t and variance $V\sigma^2$, where M_t and V are given by:

$$M_t = \frac{d}{1-\rho}\left\{(L+1) - \frac{\rho(1-\rho^{L+1})}{1-\rho}\right\} + \frac{\rho(1-\rho^{L+1})}{1-\rho} Y_t, \tag{3.8}$$

$$V = \frac{1}{(1-\rho)^2}\left\{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2 + \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}\right\}. \tag{3.9}$$

Observe that V is independent of t and it is increasing in l, L, and ρ for any $\rho \geq 0$. In this case, the manufacturer's optimal order-up-to level with no information sharing, denoted by T_t^*, is given by:

$$T_t^* = M_t + K\sigma\sqrt{V}, \tag{3.10}$$

with $K = \Phi^{-1}[P/(P + H)]$ for the standard normal distribution Φ.

3.2.2. Information Sharing. With information sharing, the manufacturer now knows both the retailer's order quantity Y_t and the error term ϵ_t (through the sharing of information about D_t) when he determines the order-up-to level T_t at the end of period t. It follows from (3.7) that the manufacturer would treat $\mathcal{B}_t$, the manufacturer's total shipment quantity over the manufacturer's lead time, as having a normal distribution F_t' with mean M_t' and variance $V'\sigma^2$, where M_t' and V' are given by:

$$M_t' = M_t - \frac{\rho(1-\rho^{L+1})(1-\rho^{l+1})}{(1-\rho)^2}\epsilon_t, \tag{3.11}$$

$$V' = \frac{1}{(1-\rho)^2}\left\{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2\right\}. \tag{3.12}$$

Again, observe that V' is also independent of t and is increasing in l, L, and ρ for any $\rho \geq 0$. In addition, note from (3.9) and (3.12) that $V' \leq V$. Thus, information sharing would reduce the variance of the total shipment quantity over the manufacturer's leadtime L. In this case, the manufacturer's optimal order-up-to level, denoted by T_t^*, is given by:

$$T_t^* = M_t' + K\sigma\sqrt{V'}. \tag{3.13}$$

Let us consider the special case in which $\rho = 0$. When there is no information sharing, it can be easily seen from (3.8), (3.9), and (3.10) that $M_t = (L + 1)d$, $V = (L + 1)$, $T_t^* = (L + 1)d + k\sigma\sqrt{L+1}$. Similarly, when there is information sharing, it can be easily seen from (3.11), (3.12), and (3.13) that $M_t' = (L + 1)d$, $V' = (L + 1)$, $T_t^* = (L + 1)d + k\sigma\sqrt{L+1}$. Thus, when $\rho = 0$, the manufacturer's order-up-to order quantity remains the same regardless of whether the manufacturer has information about the demand or not. This implies that, when $\rho = 0$, information sharing does not change the manufacturer's ordering decision. As such, information sharing does not provide benefit to the manufacturer when the autocorrelation coefficient $\rho = 0$.

4. Benefits of Information Sharing

In the last section, we have computed the means and the variances of the manufacturer's order quantity for two cases (no information sharing, and with information sharing). We now utilize the means and the variances of the manufacturer's order quantity for these two cases to evaluate the inventory reduction and cost savings associated with information sharing. First, as a consequence of the assumption that the manufacturer is responsible for the expedite cost to ensure reliable supply of products to the retailer, the retailer's cost would not be affected by information sharing. Thus, we shall focus on the impact of information sharing to the manufacturer.

In this section, we show that information sharing provides benefits to the manufacturer in two ways: (1) inventory reduction; and (2) expected cost reduction. In addition, these benefits are substantial when the underlying demand is autocorrelated and the autocorrelation coefficient is significantly (ρ is large) or highly variable (σ is large). As we shall see, these benefits of information sharing to the manufacturer can be captured nicely by the term $\sigma(\sqrt{V} - \sqrt{V'})$, where V and V' are given in (3.9) and (3.12), respectively. Both V and V' are independent of σ. In addition, we shall use the following property:

Proposition 1. *When $\rho \geq 0$, we have: (A) $\sqrt{V} - \sqrt{V'}$ is increasing in ρ, and (B) $\sqrt{V} - \sqrt{V'}$ is increasing in l.*

Proof. All proofs are given in the Appendix.

The above proposition does not hold for the case when $\rho < 0$. However, we have observed that the sales pattern of most products tend to have $\rho > 0$. First, as reported in Lee et al. (1997a), it is common to have $\rho \geq 0$ in the high-tech industry. Second, we have used panel data to examine the weekly sales pattern of 165 SKUs at a supermarket over a two-year period (June 1991–June 1993).[7] This supermarket is located in a metropolitan area in the United States and it is advertised explicitly as an EDLP (everyday low price) store. By performing the Durbin-Watson test, we have learned that the sales pattern of 150 SKUs (out of 165 SKUs) are significantly autocorrelated at 0.01 significant level. In addition, all of these 150 SKUs have positive autocorrelation coefficients ρ that vary from 0.26 to 0.89.[8] Finally, Erkip et al. (1990) have also found that the demands of consumer products are often correlated over time, with ρ as high as 0.7. Given this compelling empirical evidence, we shall restrict our attention to the case in which $\rho \geq 0$ for the remainder of this paper.

4.1. Inventory Reduction

While it is difficult to obtain exact expressions for the average inventory levels when the underlying demand process is autocorrelated over time, we shall use an approximation for the average on-hand inventory. Better approximation that utilizes the standard loss function can be obtained by other methods presented elsewhere (see, for example, Zipkin 1995). However, these methods do not result in simple close-form

[7] For each of the 33 (food and nonfood) categories in the data set, we select the top 5 best selling SKUs. Hence, we analyze the sales pattern for $33 \times 5 = 165$ SKUs altogether. We are grateful to Professor David Bell for providing us with the data. The data used here represent a portion of the data from Information Resources, Inc. Details are available upon request.

[8] Among the 33 (food and nonfood) categories that contain 5 SKUs in each category, we found that the barbecue sauce category has the highest average autocorrelation coefficient of 0.66, while the cat food category has the lowest autocorrelation coefficient of 0.17. The positive autocorrelated sales pattern is due to the repeat purchasing behavior of most shoppers. Our data analysis suggests that shoppers tend to purchase the same flavor of barbecue sauce over time. However, most shoppers tend to purchase different flavors of cat food, partly due to the "variety seeking" behavior of most cats. The repeat purchasing behavior has been studied in the marketing literature (see, for example, Guadagni and Little 1983). The brand choice model developed by Guadagni and Little (1983) assumed that a shopper will purchase the SKU (or the brand) that yields the highest utility value. In addition, after the shopper purchased a particular SKU, it is assumed that the shopper will derive additional utility value from purchasing this particular SKU in the future. This additional utility value serves as the reinforcement for repeat purchasing of the same SKU over time, which generates a positive correlated sales pattern.

expressions. Because the focus of our paper is to analyze the managerial implications of information sharing, we shall use the following approximation for the average on-hand inventory. As discussed on page 295, Silver and Petersen (1985), for any order-up-to T_t system with Y_t being the "demand" in period t, and $\sum_{i=1}^{L+1} Y_{t+i}$ being the total "demand" from period $t + 1$ to period $t + L + 1$, the average (on-hand) inventory level can be approximated by

$$\left\{ T_t - E\left(\sum_{i=1}^{L+1} Y_{t+i} \right) + E(Y_t)/2 \right\}. \tag{4.1}$$

The approximation (4.1) is usually excellent when the stockout cost is significantly higher than the inventory holding cost, although there has not been a systematic and comprehensive numerical analysis to determine what constitutes "significantly higher."

To develop the approximate expression for the manufacturer's average (on-hand) inventory, we utilize the recursive relationship of Y_{t+1} in (3.6) to show that $\lim_{t\to\infty} E(Y_t) = d/(1 - \rho)$ and that $\lim_{t\to\infty} E(\sum_{i=1}^{L+1} Y_{t+i}) = (L + 1)d/(1 - \rho)$. Combining this observation and the fact that $E(\epsilon_t) = 0$, it can be seen from (3.10) and (3.13) that: $\lim_{t\to\infty} E_{Y_t}E_{\epsilon_t}(T^*_{t+1}) = (L + 1)d/(1 - \rho) + K\sigma\sqrt{V}$ and that $\lim_{t\to\infty} E_{Y_t}E_{\epsilon_t}(T^*_{t+1}) = (L + 1)d/(1 - \rho) + K\sigma\sqrt{V'}$. It follows from (4.1) that the manufacturer's average (on-hand) inventory levels when there is no information sharing, and when there is information sharing, can be approximated respectively by:

$$I = \frac{d}{2(1 - \rho)} + K\sigma\sqrt{V}, \tag{4.2}$$

$$I' = \frac{d}{2(1 - \rho)} + K\sigma\sqrt{V'}. \tag{4.3}$$

$V' \leq V$, therefore, $I' \leq I$. This implies that the approximated manufacturer's average (on-hand) inventory can be reduced as a result of information sharing. In addition, observe that:

$$I - I' = K\sigma\{\sqrt{V} - \sqrt{V'}\}. \tag{4.4}$$

This observation and Proposition 1 imply that information sharing results in inventory reduction for the manufacturer, especially when demand is highly and positively correlated or highly variable, or when the lead time from the manufacturer to the retailer is long. For the case when the autocorrelation coefficient $\rho = 0$, let us recall from §3 that $V = L + 1$ and $V' = L + 1$. In this case, information sharing does not lead to inventory reduction. This is mainly because, when $\rho = 0$, the retailer orders every period according to the observed demand. As such, the manufacturer gets the information about the demand via the retailer's order quantity instead of via the formal mechanism for obtaining the demand information.

In addition, let us consider the percentage of inventory reduction from information sharing, denoted by $\Delta I = (I - I')/I$. It follows from (4.2) and (4.3) that:

$$\Delta I = \frac{\left(1 - \sqrt{\frac{V'}{V}}\right)}{\frac{d}{2K\sigma(1 - \rho)\sqrt{V}} + 1}. \tag{4.5}$$

We can obtain the following result about ΔI:

PROPOSITION 2. *For $\rho > 0$,*
(a) ΔI is increasing in ρ;
(b) ΔI is increasing in σ/d; and
(c) ΔI is increasing in K.

The above proposition shows that the percentage of inventory reduction from information sharing ΔI is increasing in ρ for $\rho > 0$. Next, ΔI is also increasing in σ/d; i.e., the coefficient of variation of the underlying demand. Thus, information sharing results in higher percentage of inventory reduction for the manufacturer when the underlying demand is highly uncertain. Furthermore, since K is increasing in $P/(P + H)$, ΔI is also increasing in $P/(P + H)$. Thus, when the shortage cost P is high relative to holding cost H, information sharing also results in higher percentage of inventory reduction.

4.2. Expected Cost Reduction

We now develop the expressions for manufacturer's expected inventory holding and shortage cost for two cases (no information sharing, and with information sharing). In preparation, let us define a term that will be helpful. Let $L(x)$ be the right loss function for the standard normal distribution, where:

$$L(x) = \int_x^\infty (z - x)d\Phi(z), \tag{4.6}$$

and $\Phi(z)$ is the standard normal probability distribution. By considering the first and second derivatives of $L(x)$, it is easy to show that:

LEMMA 2. *The loss function $L(x)$ is decreasing and convex in x. In addition, suppose that $K = \Phi^{-1}(P/(P + H))$. Then $(P + H)L(x) + Hx \geq (P + H)L(K) + HK\ \forall x \geq K$.*

To simplify the exposition, we first develop the expression for the manufacturer's expected inventory holding and shortage costs when there is information sharing. First, when there is information sharing, the manufacturer knows the value of the error term ϵ_t and knows that the total shipment quantity over the manufacturer's lead time, $(\mathcal{B}_t|\epsilon_t)$, given in (3.7), is normally distributed with a mean M'_t and a standard deviation $\sigma\sqrt{V'}$. In this case, the manufacturer would choose the optimal order-up-to level $T^*_t = M'_t + K\sigma\sqrt{V'}$ that minimizes the expected total inventory holding and shortage cost with respect to the true distribution F'_t. Because the total order is normally distributed, we can express C'_t, the manufacturer's expected holding and shortage costs with information sharing incurred in period $t + L + 1$, as:

$$\begin{aligned} C'_t &= E_{\epsilon_t}\left\{ P \int_{T^*}^{\infty} (x - T^*_t)dF'_t(x) \right. \\ &\qquad \left. + H \int_{-\infty}^{T^*} (T^*_t - x)dF'_t(x) \right\} \\ &= \sigma\sqrt{V'}[(H + P)L(K) + HK]. \end{aligned} \tag{4.7}$$

Next, consider the case in which there is no information sharing. The manufacturer has no information about the value of ϵ_t and the manufacturer would treat the total shipment quantity over the manufacturer's lead time, $(\mathcal{B}_t|\epsilon_t)$ (given in (3.7)), as having a distribution F_t (even though the "true" distribution is F'_t). In this case, the manufacturer thinks that the total order is normally distributed with a mean M_t and a standard deviation $\sigma\sqrt{V}$ and would choose the order-up-to level $T^*_t = M_t + K\sigma\sqrt{V}$ that minimizes the expected inventory holding and shortage cost with respect to the distribution F_t. The manufacturer thinks that the order-up-to level T^*_t is K standard deviations above the mean value of the total order over the lead time. However, in reality, since the "true" distribution of the total order over the leadtime is F'_t that has a "true" mean M'_t and a "true" standard deviation $\sigma\sqrt{V'}$, T^*_t is actually $\hat{K}$ "true" standard deviations above the "true" mean value, such that $T^*_t = M'_t + \hat{K}\sigma\sqrt{V'}$, where

$$\hat{K} = \frac{M_t - M'_t}{\sigma\sqrt{V'}} + K\frac{\sqrt{V}}{\sqrt{V'}}. \tag{4.8}$$

Because the total order is normally distributed with a mean M'_t and a standard deviation $\sigma\sqrt{V'}$, we can express C_t, the manufacturer's expected holding and shortage cost with no information sharing incurred in period $t + L + 1$, as:

$$\begin{aligned} C_t &= E_{\epsilon_t}\left\{ P \int_{T^*_t}^{\infty} (x - T^*_t)dF'_t(x) \right. \\ &\qquad \left. + H \int_{-\infty}^{T^*_t} (T^*_t - x)dF'_t(x) \right\} \\ &= E_{\epsilon_t}(\sigma\sqrt{V'}[(H + P)L(\hat{K}) + H\hat{K}]). \end{aligned} \tag{4.9}$$

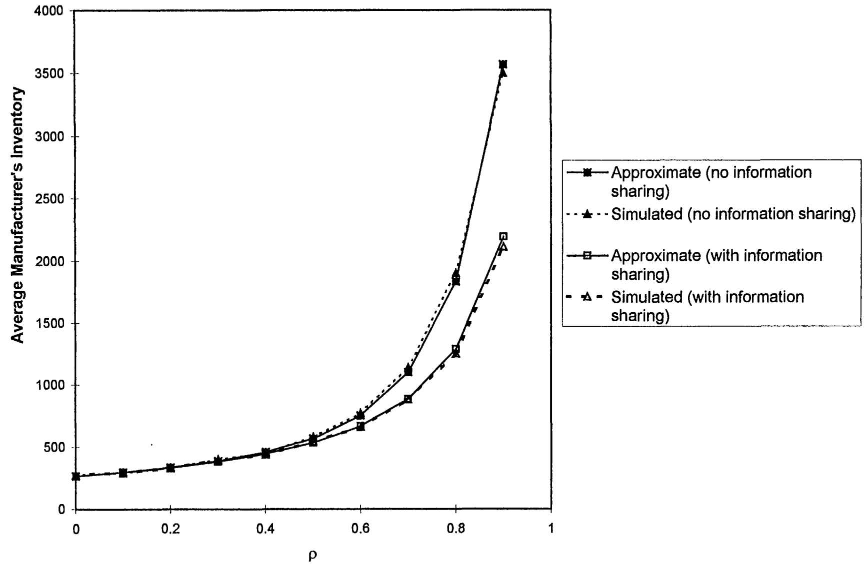
We now show that $C_t \geq C'_t$.[9] First, let us recall from Lemma 2 that the loss function is convex. Hence, we can apply Jensen's inequality to (4.9), getting:

$$C_t \geq (\sigma\sqrt{V'}[(H + P)L(E_{\epsilon_t}(\hat{K})) + H(E_{\epsilon_t}(\hat{K}))]). \tag{4.10}$$

Next, observe from (3.11) that $E_{\epsilon_t}(M_t - M'_t) = 0$ and notice from (3.9) and (3.12) that $V \geq V'$. These two observations and (4.8) imply that $E_{\epsilon_t}(\hat{K}) \geq K$. Because $E_{\epsilon_t}(\hat{K}) \geq K$, we can apply Lemma 2 to the above inequality and show that $C_t \geq C'_t$, where C'_t is given in (4.7). Therefore, we can conclude that information sharing enables the manufacturer to reduce his expected inventory holding and shortage costs.

[9] There is an alternative way to show that $C_t \geq C'_t$ holds for any general distribution. The reader is referred to Lee et al. (1996) for details.

Figure 1 Impact of ρ on Average Manufacturer's On-hand Inventory



5. Impact of Demand Process Characteristics

We now present some numerical examples to verify our analysis and to illustrate the magnitude of inventory reduction and cost savings associated with information sharing as a function of the demand process characteristics, namely, σ and ρ. Again, we focus on the case in which $\rho \geq 0$. In our example, the demand process is specified by $d = 100$. The retailer's cost parameters are given as $p = 50$, $h = 2$, and the manufacturer's cost parameters are given as $P = 25$, $H = 1$. The replenishment lead time l for the retailer equals 10, while the replenishment lead time L for the manufacturer equals 5. When we analyze the impact of ρ, we set $\sigma = 50$ and we vary ρ from 0 to 0.9. In addition, when analyzing the impact of σ, we set $\rho = 0.7$ and we vary σ from 10 to 100. Given these parameters, we first generate the random demand for 2000 consecutive time periods and we compute the simulated average actual (on-hand) inventory levels for the retailer and the manufacturer. Our intention is to examine the goodness of the approximation of the average inventory presented in the last section and to analyze the impact of information sharing on inventory reduction. Observe from (4.6), (4.7), and (4.9) that we can compute the average (inventory holding and shortage) costs per period for the manufacturer by using numerical analysis for double integrals. However, we find it more convenient to simulate the average inventory holding and shortage costs while we simulate the average inventory. This would allow us to compute the average inventory and the average cost simultaneously.

Figure 1 reports the approximate (based on (4.2) and (4.3)) and the simulated average manufacturer's inventory when ρ varies from 0 to 0.9 (with $\sigma = 50$), with and without information sharing. We observed that the simulated average manufacturer's inventory is very close to the approximate average manufacturer's inventory (within 5%), and that the average manufacturer's inventory increases as ρ increases. Figure 3 reports the approximate and the simulated percentage of inventory reduction from information sharing ΔI when ρ varies from 0 to 0.9 (with $\sigma = 50$). Figures 1 and 3 suggest that information sharing enables the manufacturer to reduce average inventory, and this inventory reduction, both in absolute terms or as a percentage of inventory, is greater when ρ is larger. The phenomena depicted in Figures 1 and 3 are consistent with the analytical findings presented in §4, that may be explained as follows. When ρ is large, current demand information is very valuable for predicting future demands, and hence, information sharing provides greater inventory reduction. It is

Figure 2 **Impact of ρ on Average Cost**

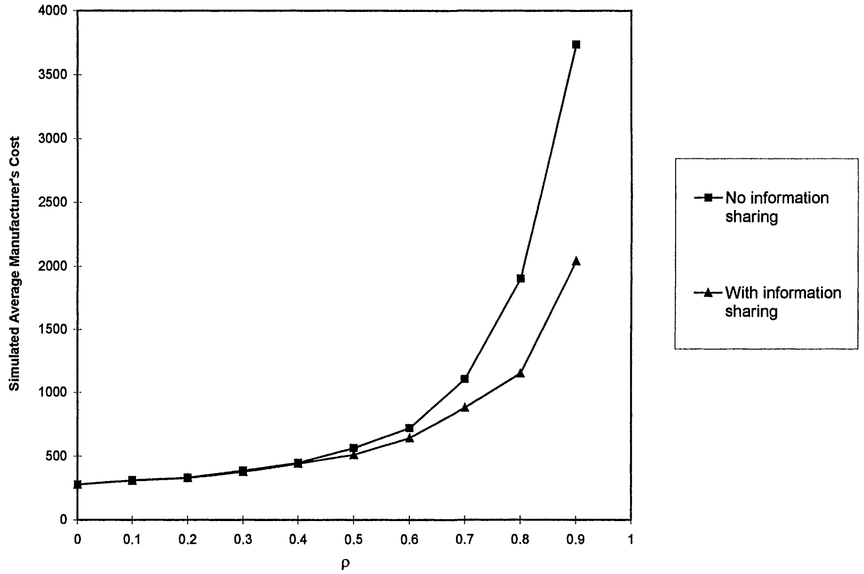
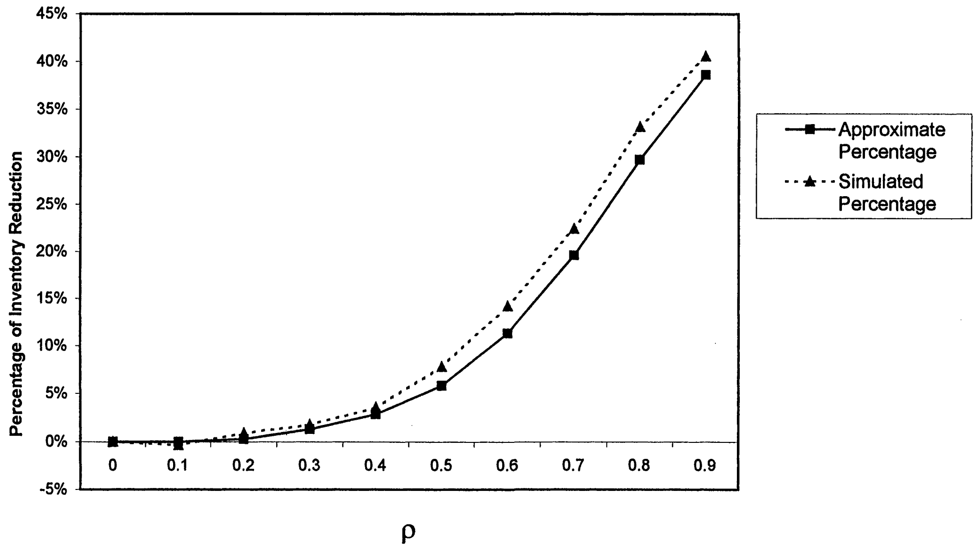


Figure 3 **Impact of ρ on Percentage of Inventory Reduction from Information Sharing**



interesting to note that, under the *i.i.d.* demand assumption used by most previous studies, i.e., $\rho = 0$, there is no value of information sharing. Our model seems to suggest that when the underlying demand process is more complex than the *i.i.d.* case, the improvement in forecasting from information sharing is of greater value. Stationary demand may therefore be insufficient to capture the benefits in a high-tech industry or the grocery industry where autocorrelated demand is prevalent. Figure 2 shows a similar observation for the impact of ρ on the (simulated) average cost with and without information sharing.

Figures 4 and 5 illustrate the simulated average inventory and the simulated actual average cost for the manufacturer when we vary σ from 10 to 100 (with $\rho = 0.7$). Figure 6 shows the impact of σ on the

Figure 4 **Impact of σ on Average Manufacturer's On-hand Inventory**

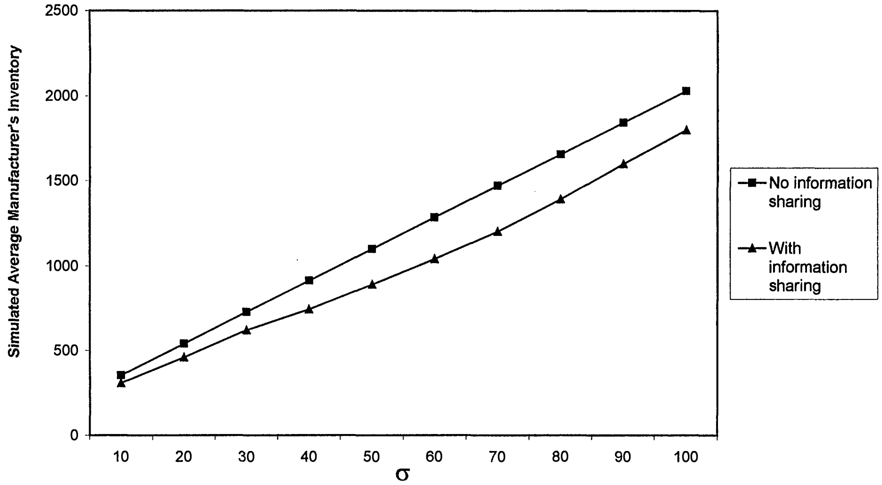
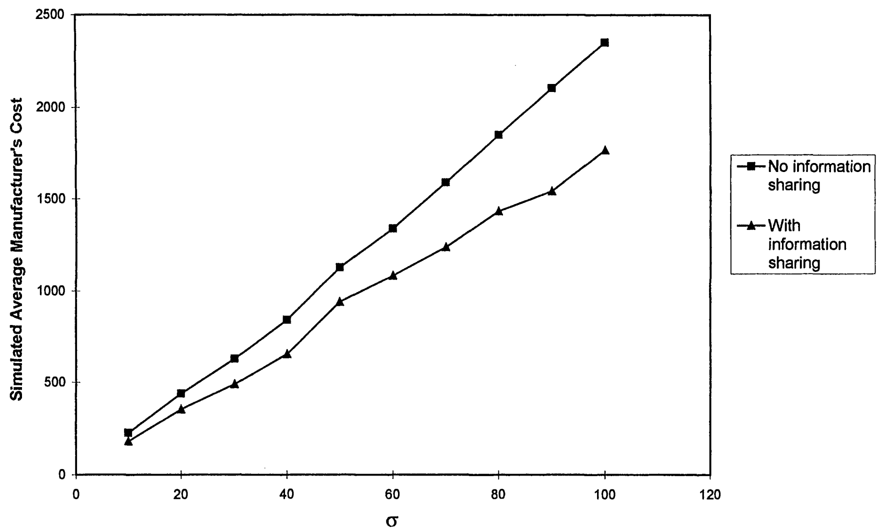


Figure 5 **Impact of σ on Average Cost**

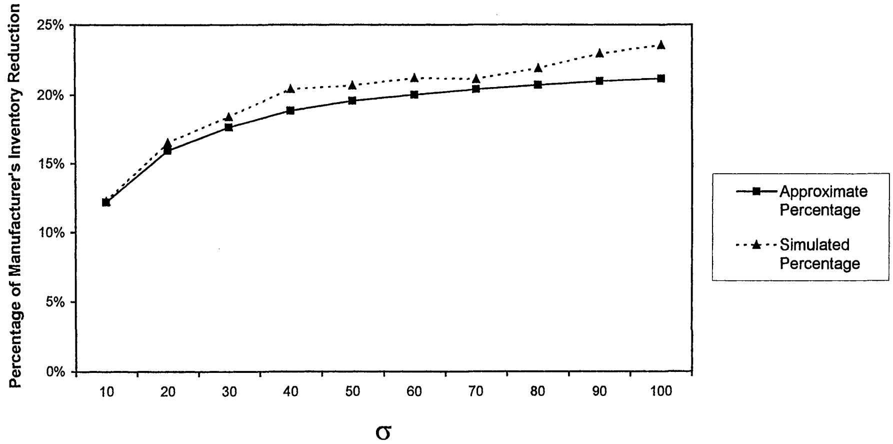


percentage of inventory reduction resulting from information sharing. As shown in Figure 5, the manufacturer's average cost increases as σ increases. In addition, Figures 4, 5, and 6 hint that information sharing provides inventory reduction and cost savings to the manufacturer, and these savings can be very substantial when σ is large.

Our numerical analysis thus far suggests that the total system benefits a great deal from information sharing when ρ is large. These benefits are in the form of reduction in inventory and in inventory holding and shortage costs for the manufacturer. In order to entice the retailer to share his demand information with the manufacturer, the manufacturer would need to provide incentives to the retailer.[10] We observe two common incentive schemes

[10] The issue of incentive alignment between the manufacturer and the retailer is beyond the scope of this paper. However, there is a new stream of innovative research work that examines the issues of

Figure 6 **Impact of σ on Percentage of Manufacturer's Inventory Reduction from Information Sharing**



in practice. The first incentive is a financial scheme that aims to reduce the retailer's variable cost. These financial incentives include price reduction, better return policy, or better payment terms, etc. The second incentive is an operational scheme that aims to reduce retailer's overhead, processing, and inventory costs. One of the operational schemes is the Vendor Managed Inventory (VMI) program that requires the manufacturer to handle the replenishment process for the retailer. Another operational scheme is to reduce the replenishment leadtime for the retailer (i.e., reduce l) so as to reduce retailer's inventory cost.

6. Impact of Lead Time

As discussed in the last section, the retailer may ask the manufacturer to reduce the replenishment lead time l in return for sharing his demand information. However, reducing replenishment lead time l could affect the manufacturer's logistic, inventory holding, and shortage costs.[11]

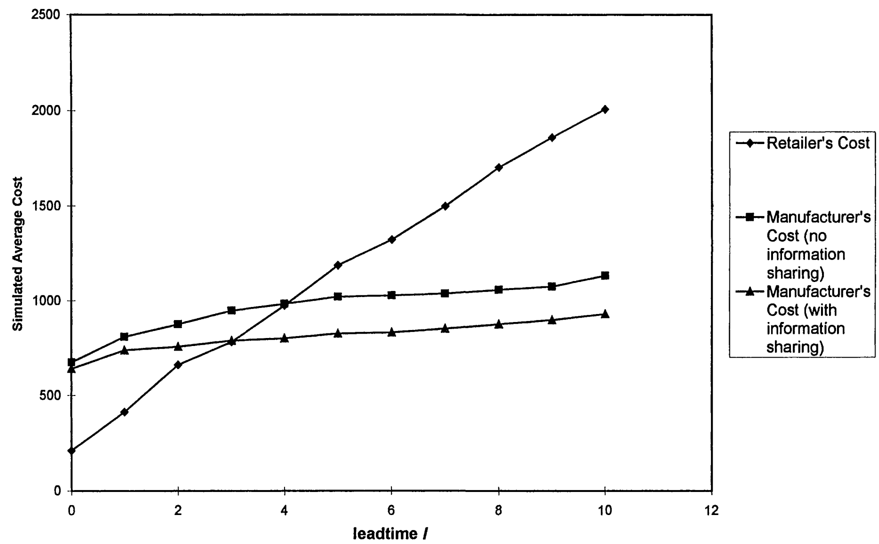
While our assumption is that the manufacturer guaranteeing reliable supply to the retailer results in information sharing being beneficial to the manufacturer only, reducing replenishment lead time l could benefit both the retailer and the manufacturer. We proceed to examine the impact of lead time l on the retailer's and the manufacturer's average inventory levels. First, by using an approximation similar to (4.2) for the retailer's inventory, it is easy to see that the retailer's inventory is a function of v, which in turn is increasing in l when $\rho \geq 0$. Hence, the approximated average inventory for the retailer is increasing in l for $\rho \geq 0$. Next, the manufacturer's inventory levels, as given by (4.2) and (4.3), are both increasing in l because both V and V' are increasing functions of l when $\rho \geq 0$. Hence, reduction in l would reduce both the retailer's and the manufacturer's inventory levels. From (4.4) and Proposition 1, we also observe that the reduction in the manufacturer's inventory due to information sharing is greater when the replenishment lead time l is larger. Our numerical example corroborates this analytical result. (Figure not shown.)

To analyze the impact of lead time l on the retailer's and the manufacturer's cost, let us consider the following numerical example with the parameters being the same as presented in the last section. However, we fix the value of $\rho = 0.7$, $\sigma = 50$, and $L = 5$, and we vary

incentive alignment between the manufacturer and the retailers. The reader is referred to Cachon and Zipkin (1999), Chen et al. (1997), and Lee and Whang (1999).

[11] Manufacturer's logistic cost could increase because reducing replenishment lead time may require the manufacturer to ship in smaller quantity more frequently. However, logistic inefficiency can be ameliorated. For example, Nabisco works with other manufacturers to make more frequent Full-Truck-Load shipments to Hanaford Brothers, while the frequency of replenishments can be increased for all manufacturers (see GMA 1996).

Figure 7 Impact of Leadtime *l* on Average Cost



l from 0 to 10. Figure 7 reports the simulated average costs for the retailer and the manufacturer when l varies from 0 to 10.

Figure 7 seems to suggest that the retailer's average cost decreases sharply, while the manufacturer's average cost declines slightly as l is shorter. Hence, long lead time l also hurts the manufacturer, as the retailer's order pattern is also more erratic. Moreover, Figure 7 hints that information sharing provides additional cost savings to the manufacturer; however, this savings does not vary much with respect to l.

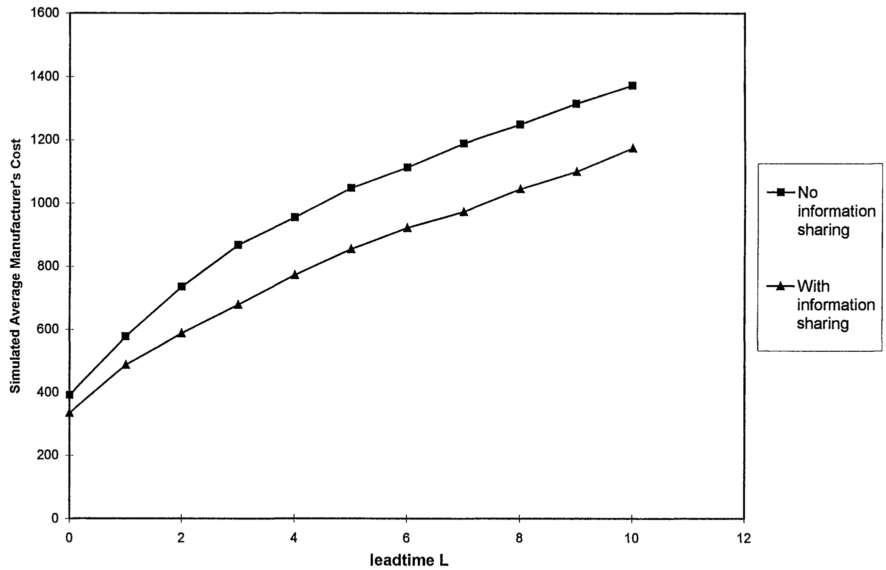
We also examine the impact of lead time L on the benefits of information sharing. This time, we set $l = 5$ and vary L from 0 to 10. Since the retailer's ordering decision as modeled in §3.1 is independent of the manufacturer's replenishment lead time L, the retailer's cost would not be affected by the manufacturer's replenishment lead time L.

Figure 8 suggests that information sharing provides relatively small cost savings to the manufacturer when L is small, but relatively large cost savings to the manufacturer when L is large. This can be explained as follows. When L is small, the manufacturer can meet and react quickly to the retailer's orders with a small amount of inventory. Thus, demand information seems to be less critical to the manufacturer when L is small.

Figure 7 seems to imply the following points. While the retailer gets no direct benefit from information sharing, the retailer obtains substantial cost savings and inventory reduction from lead time reduction. Reducing l would reduce the manufacturer's average cost only slightly and therefore by itself is not enough of an incentive for the manufacturer to invest in lead time reduction. However, this may be a means to entice the retailer to share demand information, which benefits the manufacturer. In other words, information sharing alone will benefit the manufacturer only and lead time reduction alone will benefit the retailer primarily. However, both partners may obtain benefits when information sharing and lead time reduction are implemented together. This may explain why some companies in industries such as computer, apparel, grocery, and food service develop programs that call for both information sharing and replenishment lead time reduction.[12] Finally, as reported in Figure 8, information sharing may enable the manufacturer to obtain larger cost savings when L is reasonably large. This implies that when L is large, the

[12] These programs include Quick Response (QR) for the apparel industry, Efficient Consumer Response (ECR) for the grocery industry, and Efficient Food service Response (EFR) for the food service industry (see Hammond (1993) for a discussion of QR, Kurt Salmon Associates (1993) for a discussion of ECR, and Troyer (1996) for a discussion of EFR).

Figure 8 Impact of Leadtime L on Average Cost



manufacturer may be more eager to provide the retailer with stronger incentive for obtaining demand information.

7. Discussion

This paper attempts to quantify the benefits of information sharing and to identify the drivers that have significant impacts. Under our assumption that the manufacturer bears the full cost to guarantee reliable supply to the retailer, the retailer obtains no direct benefits from information sharing alone. However, our analytical and numerical analyses show that the manufacturer can obtain inventory reduction and cost reduction with information sharing. We show that the characteristics of the demand process and the replenishment leadtime have significant impact on the benefits of information sharing to the manufacturer. Specifically, the manufacturer obtains larger reductions (in terms of average inventory and average cost) when the underlying demand is highly correlated over time, highly variable, or when the lead time is long. These findings provide valuable insights to the retailer and manufacturer when evaluating information sharing programs.

There are many open research issues that remain to be examined. First, while our model focuses on a single retailer, exactly the same approach can be used to analyze the benefit of information sharing with multiple retailers. For the case in which the demand of each retailer is an AR(1) process and the demand processes of different retailers are identical and independent, we can extend our analysis to show that the reduction of the manufacturer's average inventory is increasing in the square root of the number of retailers, while the reduction of average manufacturer's cost is increasing linearly in the number of retailers who share their demand information with the manufacturer. This may explain why it is quite common for a manufacturer to ask multiple retailers to share their demand information. (The reader is referred to Lee et al. (1996) for details.)

Next, while our model focuses on information sharing, we observe that some retailers are pushing for the manufacturer to participate in the VMI program. Such a program would require the manufacturer to monitor the retailer's inventory and to schedule replenishment deliveries to the retailer. Hence, VMI programs resemble centralized control systems, and it is of interest to analyze the relative benefits compared to information sharing as studied in this paper.[13]

[13] The authors would like to thank Mr. Juin-Kuan Chong for his technical assistance. Helpful comments were also received from seminar participants at the Chinese University of Hong Kong, Hong Kong University of Science and Technology, National University of

Singapore, University of California at Berkeley, University of Pennsylvania, and from conference participants at the INFORMS San Diego meeting in 1997. This research was partially supported by the Global Supply Chain Management Forum at Stanford University, the UCLA committee, on Research Grant #92, the UCLA Center for Technology Management, UCLA James Peters Research Fellowship, and the Hong Kong Polytechnic University Research Grant #351/326.

Appendix

PROOF OF LEMMA 1. Observe from (2.1) and (3.5) that Y_t can be rewritten as:

$$Y_t = \rho^{l+2} D_{t-1} + \frac{1-\rho^{l+2}}{1-\rho}(d + \epsilon_t).$$

Since ϵ_t are *i.i.d.* $N(0, \sigma^2)$ random variables, the retailer's order quantity Y_t is normally distributed. This implies that the statement $P\{Y_t \geq 0\}$ is increasing in ρ is equivalent to $\sqrt{\mathrm{Var}(Y_t)/E(Y_t)}$ is decreasing in ρ. We shall now prove the latter statement.

By applying the fact that $\lim_{t\to\infty} E(D_{t-1}) = d/1-\rho$, $\lim_{t\to\infty} \mathrm{Var}(D_{t-1}) = \sigma^2/1-\rho^2$, and that $\epsilon_t = N(0, \sigma^2)$, it is easy to show from Y_t above that, as $t \to \infty$,

$$E(Y_t) = \frac{d}{1-\rho}$$

$$\mathrm{Var}(Y_t) = \frac{\sigma^2}{(1-\rho)^2}\left(\frac{\rho^{2l+4}(1-\rho)}{1+\rho} + (1-\rho^{l+2})^2\right).$$

Therefore, as $t \to \infty$,

$$g(\rho) = \frac{\mathrm{Var}(Y_t)}{E(Y_t)^2} = \frac{\sigma^2}{d^2}\left(\frac{\rho^{2l+4}(1-\rho)}{1+\rho} + (1-\rho^{l+2})^2\right).$$

We can obtain from direct differentiation that $g'(\rho) < 0$ for $\rho > 0$. This show that $\mathrm{Var}(Y_t)/E(Y_t)^2$ is decreasing in ρ, and we have proved the second statement. □

PROOF OF PROPOSITION 1. *We first introduce two useful lemmas. In preparation, consider* Δ *as a function of* ρ *and* l, where $\Delta = 1 - \sqrt{V_2/V_1}$, and V_1 and V_2 are two arbitrary functions of ρ and l.

LEMMA 3. *Suppose that $V_1 \geq V_2 \geq 0$ for any $\rho > 0$, and that Δ and V_1 are both increasing in ρ for any $\rho > 0$. Then $\sqrt{V_1} - \sqrt{V_2}$ is increasing in ρ for any $\rho > 0$.*

LEMMA 4. *Suppose that $V_1 \geq V_2 \geq 0$ and $V_1 = V_2 + X$. Then Δ is increasing in ρ for $\rho > 0$ if and only if $V_2(\partial X/\partial\rho) - X(\partial V_2/\partial\rho)$ is nonnegative.*

PROOF. Both lemmas can be proven by considering the derivative of Δ with respect to ρ.

PROOF OF PROPOSITION 1, PART (A). $\sqrt{V} - \sqrt{V'}$ is increasing in ρ for any $\rho > 0$.

Let $V_1 = V$, $V_2 = V'$. Then $V_1 = V_2 + X$, where

$$X = \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^4}. \tag{A1}$$

For $1 > \rho > 0$, it is easy to check from (3.9) and (3.12) that $V_1 \geq V_2 \geq 0$ and $X \geq 0$, and that V_1 is increasing in ρ for $\rho > 0$. By applying Lemmas 3 and 4, we can prove part (A) by showing that $V_2(\partial X/\partial\rho) - X(\partial V_2/\partial\rho)$ is nonnegative for $\rho > 0$. Substitute (3.9), (3.12), and (A1) into $V_2(\partial X/\partial\rho) - X(\partial V2/\partial\rho)$ and then simplify some terms to obtain:

$$V_2\frac{\partial X}{\partial\rho} - X\frac{\partial V_2}{\partial\rho}$$

$$= 2\frac{1}{(1-\rho)^3}\rho\left(\sum_{k=0}^{L}\rho^k\right)\left(\sum_{j=0}^{l}\rho^j\right)\left(\sum_{i=0}^{L}(1-\rho^{l+2+i})Z_i\right), \tag{A2}$$

where

$$\begin{aligned} Z_i &= (l+2+i)(1-\rho)\rho^{l+2+i}\left(\sum_{k=0}^{L}\rho^k\right)\left(\sum_{j=0}^{l}\rho^j\right) \\ &\quad + (1-\rho^{l+2+i})\left\{\left(\sum_{k=0}^{L}\rho^k\right)\left(\sum_{j=0}^{l}\rho^j\right) - (l+1)\rho^{l+1}\left(\sum_{k=0}^{L}\rho^k\right)\right. \\ &\quad \left. - (L+1)\rho^{L+1}\left(\sum_{j=0}^{l}\rho^j\right)\right\}. \end{aligned} \tag{A3}$$

It remains to show that $Z_i \geq 0$ for each i, $i = 0, 1, \ldots, L$. Consider three cases: (1) $i \leq L-2$; (2) $i = L-1$; and (3) $i = L$. (To simplify the exposition, we shall present the case $l = 0$. The same approach can be used to prove the general case $l > 0$. Details are available upon request.)

Case 1. $i \leq L-2$. Since $l = 0$, (A3) is reduced to:

$$\begin{aligned} Z_i &= (2+i)(1-\rho)\rho^{2+i}\left(\sum_{k=0}^{L}\rho^k\right) \\ &\quad + (1-\rho^{2+i})\left\{\left(\sum_{k=0}^{L}\rho^k\right) - \rho\left(\sum_{k=0}^{L}\rho^k\right) - (L+1)\rho^{L+1}\right\} \\ &= (2+i)(1-\rho)\left(\sum_{k=0}^{L}\rho^{k+2+i}\right) \\ &\quad + (1-\rho)\left(\sum_{j=0}^{i+1}\rho^j\right)(1-(L+2)\rho^{L+1}) \\ &= (1-\rho)\left\{(2+i)\sum_{k=i+2}^{L}\rho^k + (2+i)\sum_{k=L+1}^{L+2+i}\rho^k\right. \\ &\quad \left. + \sum_{j=0}^{i+1}\rho^j - (L+2)\sum_{k=L+1}^{L+2+i}\rho^k\right\} \\ &= (1-\rho)\left\{\sum_{j=0}^{i+1}\rho^j + (2+i)\sum_{k=i+2}^{L}\rho^k - (L-i)\sum_{k=L+1}^{L+2+i}\rho^k\right\}. \end{aligned} \tag{A4}$$

We make two observations on the bracketed sums on the right hand side of (A4). First, the power of the positive terms are of lower order (i.e., the power of ρ varies from 0 to L), while the power of the negative terms are of higher order (i.e., the power of ρ varies from $L+1$ to $L+2+i$). Second, the total number of positive terms is equal to $L+1$, while the total number of negative terms is equal to $i+2$. Because $i \le L-2$, the total number of negative terms is less than the number of positive terms. These two observations and the fact that $\rho > 0$ imply that $Z_i \ge 0$ if we can show that the sum of the coefficients associated with the positive terms is no less than the sum of the coefficients associated with the negative terms. To show this, notice from (A4) that the sum of the coefficients associated with the positive terms is equal to $(i+2)+(2+i)(L-i-1) = (L-i)(2+i)$, while the sum of the coefficients associated with the negative terms is equal to $(L-i)(2+i)$. This proves Case 1.

Case 2. $i = L-1$. Because $l = 0$, (A3) is reduced to:

$$Z_i = (L+1)(1-\rho)\rho^{L+1}\left(\sum_{k=0}^{L} \rho^k\right) + (1-\rho^{L+1})(1-(L+2)\rho^{L+1})$$

$$= (1-\rho^{L+1})\{(L+1)\rho^{L+1} + 1 - (L+2)\rho^{L+1}\} = (1-\rho^{L+1})^2 \ge 0.$$

Case 3. $i = L$. Because $l = 0$, it is easy to check from (A3) that Z_i can be expressed as:

$$Z_i = (L+2)(1-\rho)\rho^{L+2}\left(\sum_{k=0}^{L} \rho^k\right)$$

$$+ (1-\rho^{L+2})\left\{\sum_{k=0}^{L} \rho^k - \rho \sum_{k=0}^{L} \rho^k - (L+1)\rho^{L+1}\right\}$$

$$= 1 + (L+1)\rho^{L+2} - (L+2)\rho^{L+1}.$$

Consider Z_i as a function of ρ. Since $Z_i(0) = 1$, $Z_i(1) = 0$, and $(\partial Z_i/\partial \rho) \le 0$ for any $\rho > 0$. Hence, $Z_i \ge 0$ for all $0 < \rho < 1$. This completes the proof for part (A). □

Consider l as any nonnegative integer and consider the following lemmas.

LEMMA 5. *Suppose that* $V_1 \ge V_2 \ge 0$ *and suppose that* Δ *and* V_1 *are both increasing in* l, *then* $\sqrt{V_1} - \sqrt{V_2}$ *is increasing in* l.

LEMMA 6. *Suppose that* $V_1 \ge V_2 \ge 0$ *for any* $l \ge 0$, *and that* $V_1 = V_2 + X$; *then* Δ *is increasing in* l *for* $l \ge 0$ *if and only if* $V_2(l)X(l+1) - V_2(l+1)X(l) \ge 0$.

Both lemmas can be easily proven by considering the term $\Delta(l+1) - \Delta(l)$.

PROOF OF PROPOSITION 1, PART (B). $\sqrt{V} - \sqrt{V'}$ is increasing in l for any $\rho > 0$.

Let $V_1(l) = V(l)$, $V_2(l) = V'(l)$. Then $V_1(l) = V_2(l) + X(l)$, where

$$V_2(l) = \left(\sum_{j=0}^{l+1} \rho^j\right)^2 + \frac{1}{(1-\rho)^2}\sum_{j=1}^{L} (1-\rho^{L+l+3-j})^2; \tag{A5}$$

$$X(l) = \rho^2\left(\sum_{j=0}^{L} \rho^j\right)^2\left(\sum_{k=0}^{l} \rho^k\right)^2. \tag{A6}$$

It follows from Lemmas 5 and 6 that $\sqrt{V_1(l)} - \sqrt{V_2(l)}$ is increasing in l if $V_2(l)X(l+1) - V_2(l+1)X(l) \ge 0$. By substituting $V_2(l)$ from (A5) and $X(l)$ from (A6) and by eliminating some irrelevant terms, it can be shown that $V_2(l)X(l+1) - V_2(l+1)X(l) \ge 0$ if

$$\left(\sum_{j=0}^{l+1} \rho^j\right)^4 + \left(\sum_{k=0}^{l+1} \rho^k\right)^2 \frac{1}{(1-\rho)^2}\sum_{j=1}^{L} (1-\rho^{L+l+3-j})^2$$

$$- \left(\sum_{k=0}^{l+2} \rho^k\right)^2\left(\sum_{k=0}^{l} \rho^k\right)^2 - \left(\sum_{k=0}^{l} \rho^k\right)^2$$

$$\cdot \frac{1}{(1-\rho)^2}\sum_{j=1}^{L} (1-\rho^{L+l+4-j})^2 \ge 0.$$

The above inequality can be rearranged as:

$$\left\{\left(\sum_{j=0}^{l+1} \rho^j\right)^4 - \left(\sum_{k=0}^{l+2} \rho^k\right)^2\left(\sum_{k=0}^{l} \rho^k\right)^2\right\} + \left\{\frac{1}{(1-\rho)^2}\sum_{j=1}^{L}\left[(1 - \rho^{L+l+3-j})^2\left(\sum_{k=0}^{l+1} \rho^k\right)^2 - (1-\rho^{L+l+4-j})^2\left(\sum_{k=0}^{l} \rho^k\right)^2\right]\right\} \ge 0. \tag{A7}$$

It remains to show that both the first term and the second term of (A7) are nonnegative. The first term can be written as:

$$\left\{\left(\sum_{j=0}^{l+1} \rho^j\right)^2 - \left(\sum_{k=0}^{l+1} \rho^k + \rho^{l+2}\right)\left(\sum_{k=0}^{l+1} \rho^k - \rho^{l+1}\right)\right\}$$

$$\cdot\left\{\left(\left(\sum_{j=0}^{l+1} \rho^j\right)^2 + \left(\sum_{k=0}^{l+2} \rho^k\right)\left(\sum_{k=0}^{l} \rho^k\right)\right)\right\}$$

$$= \rho^{l+1}\left(\left(\sum_{j=0}^{l+1} \rho^j\right)^2 + \left(\sum_{k=0}^{l+2} \rho^k\right)\left(\sum_{k=0}^{l} \rho^k\right)\right) \ge 0. \tag{A8}$$

The second term in (A7) can be simplified as:

$$\frac{1}{(1-\rho)^2}\sum_{j=1}^{L} \rho^{l+1}(1-\rho^{L+2-j})\left[(1-\rho^{L+l+3-j})\left(\sum_{k=0}^{l+1} \rho^k\right) + (1-\rho^{L+l+4-j})\left(\sum_{k=0}^{l} \rho^k\right)\right] \ge 0.$$

This completes the proof for part (B).

PROOF OF PROPOSITION 2. (a) Assume $\rho > 0$. First, observe from (3.9) and (3.12) that the first two terms inside the brackets for V (and V'

as well) are decreasing in ρ. Second, the last term inside the bracket for V in (3.9) is increasing in ρ. Therefore, V'/V is decreasing in ρ, and the numerator $\{1 - \sqrt{V'/V}\}$ in (4.5) is increasing in ρ. Third, by noting from (3.9) that the term $(1 - \rho)\sqrt{V}$ is increasing in ρ, it is easy to see that the denominator $\{d/2K\sigma(1 - \rho)\sqrt{V} + 1\}$ in (4.5) is decreasing in ρ. Combining these observations, we can conclude that the percentage of inventory reduction from information sharing ΔI is increasing in ρ.

(b) Because V and V' are independent of d and σ, it is easy to check from (4.5) that ΔI is increasing in σ/d. This completes the proof.

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