International Journal of Production Research

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tprs20

Bullwhip effect and inventory oscillations analysis using the beer game model

Matteo Coppini [a], Chiara Rossignoli [a], Tommaso Rossi [a] & Fernanda Strozzi [a]

[a] Carlo Cattaneo, LIUC University, 21053 Castellanza, VA, Italy

Version of record first published: 09 Jun 2009.

To cite this article: Matteo Coppini, Chiara Rossignoli, Tommaso Rossi & Fernanda Strozzi (2010): Bullwhip effect and inventory oscillations analysis using the beer game model, International Journal of Production Research, 48:13, 3943-3956

To link to this article: http://dx.doi.org/10.1080/00207540902896204

International Journal of Production Research
Vol. 48, No. 13, 1 July 2010, 3943–3956

Bullwhip effect and inventory oscillations analysis using the beer game model

Matteo Coppini, Chiara Rossignoli, Tommaso Rossi and Fernanda Strozzi*

Carlo Cattaneo, LIUC University, 21053 Castellanza, VA, Italy

(*Received 12 September 2008; final version received 9 March 2009*)

This work examines the bullwhip effect generated and suffered by each level of a four-stage beer game supply chain when different demand scenarios are considered. The paper shows that the actors who generate lower bullwhip are those who suffer more from its effects. Moreover, a new definition of an inventory oscillations measure based on bullwhip definition is introduced. Finally the paper verifies that the new measure of inventory oscillations provides more information on supply chain performance than the bullwhip measure.

Keywords: Bullwhip effect; supply chain dynamics

1. Introduction

The bullwhip effect refers to the phenomenon that occurs in a supply chain when orders submitted to suppliers have a greater variability than those received from customers. This causes a distortion and amplification of demand variability moving up in the supply chain (Lee *et al*. 1997). The consequence of such orders variance increase is the need for larger stocks, extra production capacity, and more storage space (Chatfield *et al*. 2004).

The first academic description of the phenomenon is generally attributed to Forrester (1961) who explained that the bullwhip effect is due to the lack of information exchange between actors in the supply chain as well as to the existence of non-linear interactions. Over the years countless studies have been made regarding this phenomenon and the literature about it is considerable (Kahn 1987, Baganha and Cohen 1998, Lee *et al*. 1997, 2000, Metters 1997, Chen *et al*. 2000, Chatfield *et al*. 2004, Geary *et al*. 2006).

Most of these studies aimed at demonstrating the existence of the bullwhip effect and at identifying its causes or possible countermeasures. In particular, Lee *et al*. (1997, 2000) identify the main causes of the bullwhip effect in demand signal processing (i.e. incorrect demand forecasting), rationing game and lead-time, order batching and prices variations. They also indicate some countermeasures, amongst them: reduction of uncertainty along the supply chain by providing each stage with complete information on customer demand, and lead-time reduction by using EDI (electronic data interchange). Moreover they quantify the benefits on the bullwhip effect of such countermeasures for a multiple-stage supply chain with non-stationary customer demands.

Chen *et al*. (2000) measure the impact of information sharing on the bullwhip effect for a two-stage, order-up-to inventory policy-based supply chain. They demonstrate that

*Corresponding author. Email: fstrozzi@liuc.it

ISSN 0020–7543 print/ISSN 1366–588X online

DOI: 10.1080/00207540902896204
http://www.informaworld.com

the bullwhip effect can be reduced but not completely eliminated by centralising demand information. Dejonckeere *et al.* (2004) present a summary of the impact on the bullwhip of different forecasting techniques together with an order-up-to inventory policy, both with and without information enrichment, whereas Chatfield *et al.* (2004) analyse the impact of stochastic lead time and information quality on the bullwhip. They consider three different bullwhip measures:

(i) the standard deviation of the order quantities at each node,
(ii) the ratio of the order variance at node 'k' to the order variance of the customer and
(iii) the ratio of the order variance at node 'k', to the order variance at node '$k-1$'.

Different techniques have been applied to reduce the bullwhip effect and among them genetic algorithms (O'Donnell *et al.* 2006), fuzzy inventory controller (Xiong and Helo 2006), distributed intelligence (De La Fuente and Lozano 2007).

Techniques to reduce the bullwhip effect based on considering the supply chain as a dynamic system and the application of control techniques are summarised by Sarimveis *et al.* (2008). These control methodologies span from the application of a proportional control (Disney and Towill 2003, Chen and Disney 2007) to highly sophisticated techniques, such as model predictive control (Tzafestas *et al.* 1997). Finally, Strozzi *et al.* (2008) propose a new chaos theory technique that consists of measuring the divergence of the system in state space and reducing the bullwhip and the costs connected to it by reducing that divergence.

This work differs from previous studies in several ways. First, we focus on how the actor's position in the supply chain influences his responsibility in the bullwhip generation, as well as his predisposition to suffer from bullwhip. As a consequence, we analyse the bullwhip effect generated and suffered by each level of the supply chain in different customer demand scenarios when information is not shared and the order policy changes from a nervous to a calm behaviour. In particular, we measure the generated and suffered bullwhip according to a single-stage and multi-stage variance amplification model (Chatfield *et al.* 2004) and we show that supply chains are 'unfair' systems: the stages that are more responsible for the bullwhip generation are those that suffer less from it. Second, we extend the definition of bullwhip to variables other than the orders, i.e. to stock levels. In this way more significant information on supply chains performances can be obtained from the bullwhip analysis. Third, we compare the outcomes of the measure of the order oscillations, i.e. the bullwhip effect and the inventory oscillations, with the overall cost faced by each supply chain actor. The comparison shows the effectiveness of the proposed measure in depicting the overall cost trend along the supply chain.

To carry out our analysis we consider and simulate the beer game supply chain consisting of one retailer, one wholesaler, one distributor and one manufacturer. In Section 2 we describe the beer game supply chain model, the inventory policy, the forecasting technique and the order policy used. In Section 3 we demonstrate the inverse relation between generated and suffered bullwhip, whereas in Section 4 we extend the bullwhip definition to stock levels and analyse again the generated and suffered bullwhip by means of the new definition. In Section 5 we compare the previously obtained outcomes with the costs and the oscillation in the effective inventory level (i.e. the sum of inventory level and backlogs). Finally, in Section 6 we conclude with the discussion of our results.

2. The beer game supply chain model

Consider a simple supply chain (composed of one retailer, one wholesaler, one distributor and one manufacturer) in which in each period, t, an actor observes his inventory level and places an order, O_t, to the upstream supply chain stage. There is a minimum lead time between the time an order is placed and when it is received, such that an order placed at the end of period t is received at the start of period $t+3$ (if, of course, the supplier inventory is sufficient to satisfy the customer order).

The manufacturer has unlimited production capacity and each actor has unlimited storage capacity.

We consider four types of final customer demands (see Figure 1) and we use them for developing and studying four different scenarios.

2.1 *The model equations*

Each actor follows a simple order-up-to inventory policy in which the order-up-to point, Q, is:

$$Q = DINV + \beta * DSL \tag{1}$$

where $DINV$ and DSL are the desired inventory level and the desired stock in transit directed towards the respective actor and β is a parameter whose value ranges from 0 to 1 (see below for further details on β).

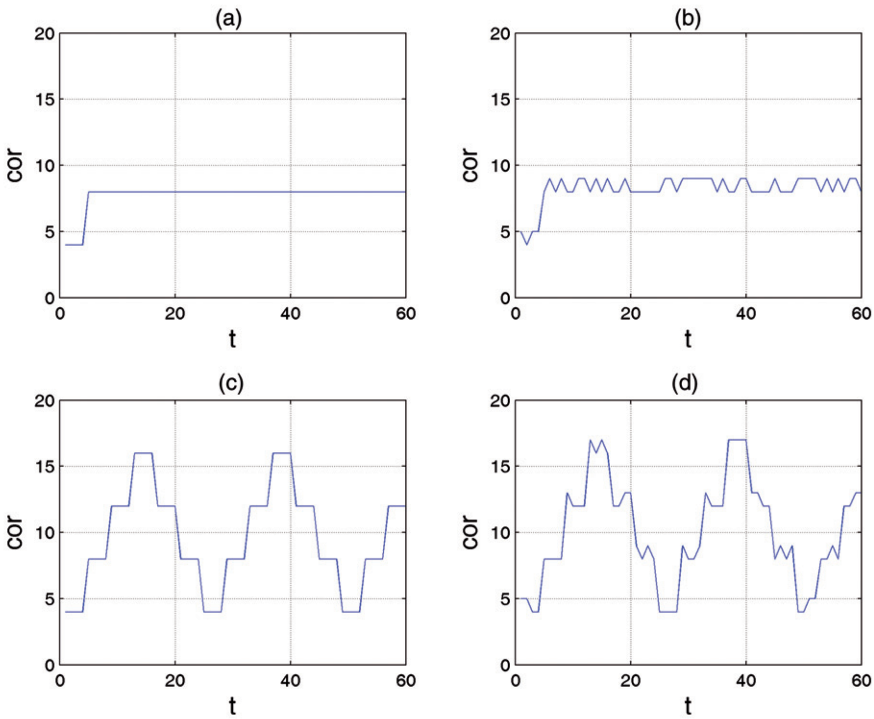

Figure 1. Types of final customer demand (COR) analysed: (a) step; (b) step with noise with variance 1; (c) cyclic; (d) cyclic with noise with variance 1.

We assume that the forecasting technique used in every stage is exponential smoothing. In particular, in each period, t, for each actor, the expected demand, ED_t, depends on the actual demand for the previous period, i.e. the incoming orders at time $t-1$, IO_{t-1}, as well as on the expected demand for the past period, ED_{t-1}. That is,

$$ED_t = \theta * IO_{t-1} + (1-\theta) * ED_{t-1} \tag{2}$$

where θ $(0 \leq \theta \leq 1)$ is the weight given to the incoming orders with respect to the expected demand.

The order O_t each actor places at time t, to the upstream supply chain stage is

$$O_t = \max\{0, O_t^*\} \tag{3}$$

where

$$O_t^* = ED_t + AS_t + ASL_t \tag{4}$$

with AS_t and ASL_t the stock and stock in transit adjustment, respectively. We can write them as

$$AS_t = \alpha_S * (DINV - INV_t + BL_t) \tag{5}$$

$$ASL_t = \alpha_{SL} * (DSL - SL_t) \tag{6}$$

where INV_t and BL_t are the inventory and backlog levels experienced by the actor at time t and SL_t is the actual stock in transit directed toward the same actor, with α_S $(0 \leq \alpha_S \leq 1)$ the stock adjustment rate and α_{SL} the stock in transit adjustment rate. Higher values of these parameters correspond to a nervous policy in which the actor quickly changes his order when the stock or the supply chain moves away from the desired value.

Since $\beta = \alpha_{SL}/\alpha_S$, Equation (4) can be rewritten as:

$$O_t^* = ED_t + \alpha_s * (Q - INV_t + BL_t - \beta * SL_t) \tag{7}$$

2.2 *The experimental campaign*

We used Matlab 7.0 to simulate the above described supply chain model. We assume that: α_S and β are the same for the four stages; $\theta = 0.25$ and $Q = 14$; the initial inventory of each actor is 12 and the simulation run length is 60 periods as in Mosekilde and Larsen (1991).

The simulation is performed according to the four types of final customer demand scenarios depicted in Figure 1 and to a single-stage and multi-stage approach. With the single-stage approach we measure the increase of bullwhip that occurs at each supply chain stage, i.e. the bullwhip generated by each stage; with the multi-stage approach we measure the bullwhip increase going upstream in the supply chain with respect to the customer demand, i.e. the bullwhip suffered by each stage.

Figure 2 synthesises the eight simulation runs performed called experimental campaigns. For each customer demand we have considered two cases: in the first, the variance amplification for each level is measured with respect to the downstream actor, while, in the second, to the final customer. For each simulation run we have recorded the incoming and outgoing orders and the inventory levels of each sector. We need these data for demonstrating the inverse relationship between generated and suffered bullwhip

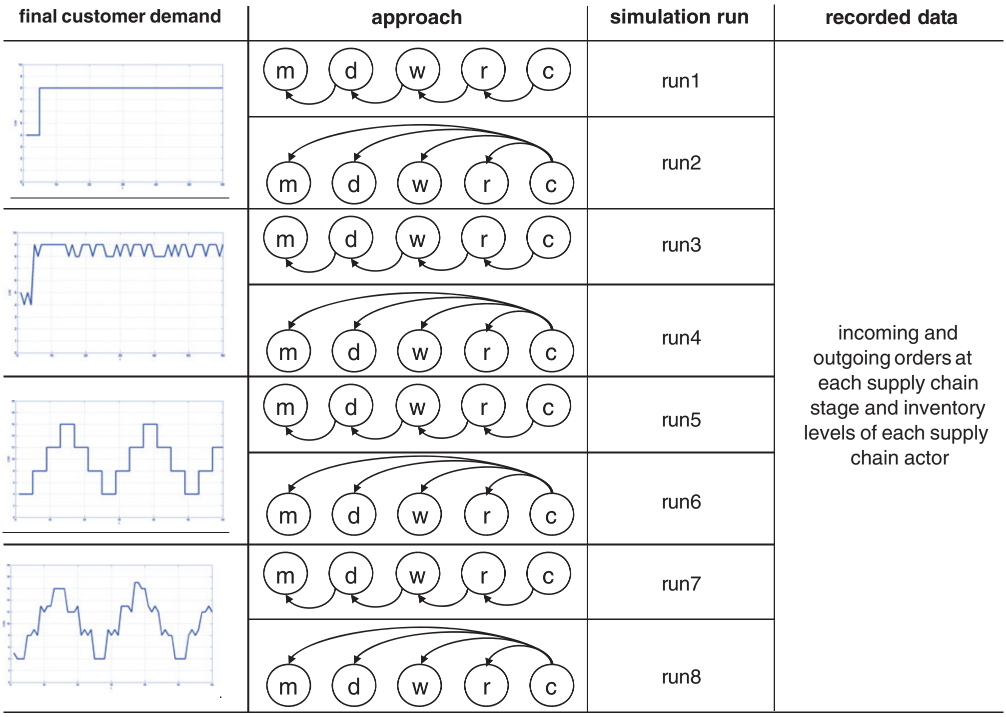

Figure 2. Synthesis of the experimental campaigns.

and for extending the traditional bullwhip definition. We have also monitored costs and inventories maximum oscillation. The costs each actor must face are calculated using Equation (8) where C_{INV} and C_{BL} are the unitary inventory and backlog costs respectively ($C_{INV}=0.50$ \$/unit, $C_{BL}=2$ \$/unit; Sterman 1989).

$$COST = \Sigma_t INV_t * C_{INV} + \Sigma_t BL_t * C_{BL} \tag{8}$$

The inventories' maximum oscillations are the maximum oscillations of the effective inventory ($INV_t - BL_t$) that occur during the 60 weeks in the whole supply chain (Caloiero *et al.* 2008).

3. Generated and suffered bullwhip analysis

According to the single-stage model the bullwhip is calculated as the ratio of the variance of orders placed by each level to the variance of orders coming from the downstream level in the supply chain. Hence the bullwhip generated by the level i (BOG_i) is

$$\frac{BOG_i = \text{Var(order placed by level `}i\text{')}}{\text{Var(order placedby level `}i-1\text{')}} \tag{9}$$

The bullwhip surfaces are generated for each customer demand scenario, each supply chain level and for variations in α_S and β, i.e. when the behaviour of the four actors in response to their own stock level and to the stock in transit directed towards them change (see Figure 3).

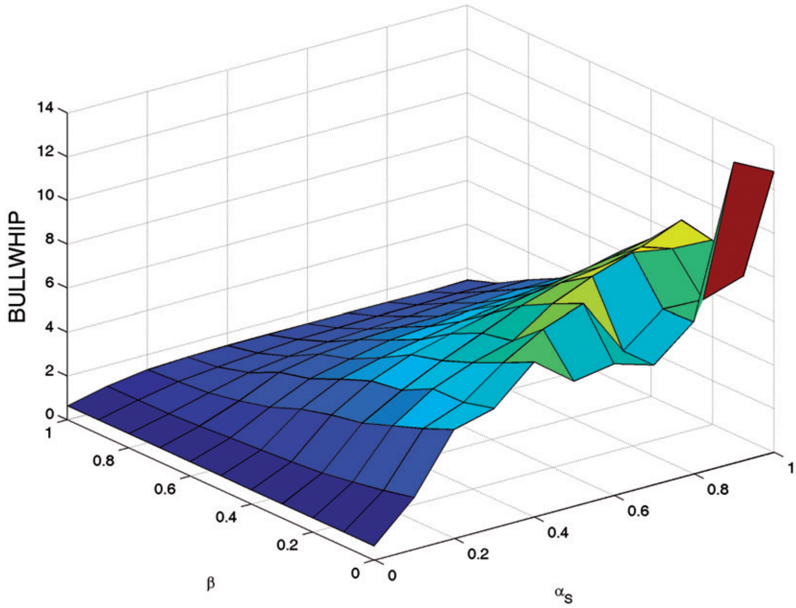

Figure 3. Surface of the bullwhip suffered by retailer in the case of multi-stage model and for customer demand given by scenario 3 plotted with respect to the α_S and β parameters.

Since we are interested in calculating how the actor's position in the supply chain influences his responsibility in the bullwhip generation (as well as his predisposition to suffer from bullwhip) and, since we do not focus on how the behaviour of each actor increases the bullwhip, we synthesise the surfaces by means of their average values.

In Figure 4 the average values of the bullwhip generated by each supply chain actor are represented for all the demand scenarios.

It is possible to observe that mean values for each of the four demand scenarios decrease, moving upstream in the supply chain. This is coherent with the bullwhip analysis results obtained by Chatfield *et al.* (2004) for a random customer demand with differing degree of communication among levels. The retailer is the one who has greater responsibility in the creation of the bullwhip, responsibility that gradually decreases going upstream in the supply chain. This may be due to the presence of multi-level inventories which tend to damp the demand fluctuations.

Concerning the bullwhip suffered by each supply chain level (BOS_i), that refers to a multi-stage model. In this case, the bullwhip is measured as the ratio between the variance of orders placed by each level to the customer order rate (COR) variance, so that

$$\frac{BOS_i = \text{Var(order placedby level `}i\text{')}}{\text{Var}(COR)} \tag{10}$$

Looking at the definitions given by Equations (9) and (10), in the case of the retailer the bullwhips generated and suffered are the same. This means that the retailer suffers only the bullwhip that he generates. Obviously, the retailer also suffers negative consequences from the bullwhip of the other levels. In fact, if the other actors make wrong demand forecasts

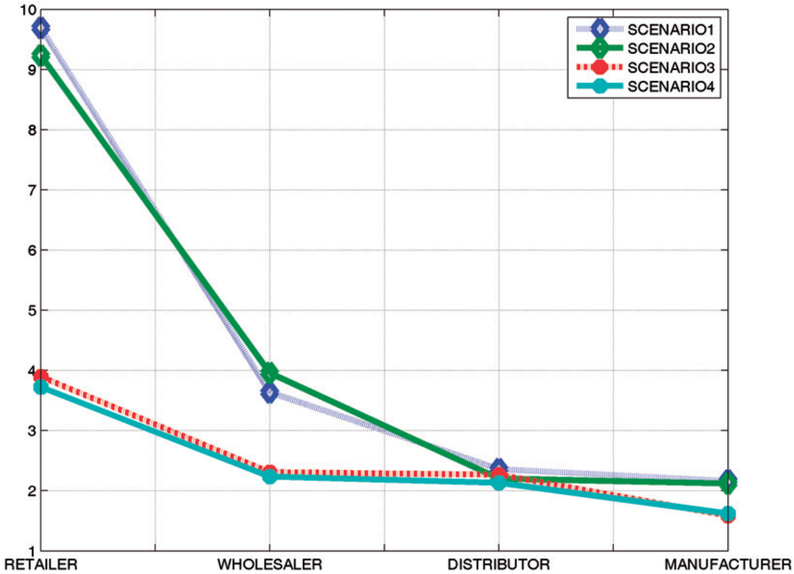

Figure 4. Decreasing of mean bullwhip values going upstream in the supply chain in the case of the single-stage model.

they cannot satisfy the retailer's demand and the retailer himself will experience an increase in backlog.

Again we consider the mean values of the bullwhip suffered by each supply chain level for the four demand scenarios. The corresponding graph is shown in Figure 5. The situation is now reversed compared with the single-stage model: average values of bullwhip grow exponentially moving upstream in the supply chain. Here the effects are more dramatically detectable in the most remote areas far from the customers, who although they are less responsible for creating this phenomenon, are the most affected because of their position in the supply chain.

4. Generated and suffered inventory oscillations analysis

In this section we define a measure of the inventory oscillations similar to the bullwhip measure for order oscillations and we repeat the analysis performed in Section 3 on the bullwhip measure. The introduction of this new definition has the objective to give some insights on the consequences that the inventory oscillations have on inventory management costs.

According to the single-stage model, the inventory oscillations generated by each level i, IOG_i can be quantified as the ratio between the variance of the inventory of the considered supply chain level to the inventory variance of the downstream level, so that

$$\frac{IOG_i = \text{Var(Inventory of level `}i\text{')}}{\text{Var(Inventory of level `}i-1\text{')}} \tag{11}$$

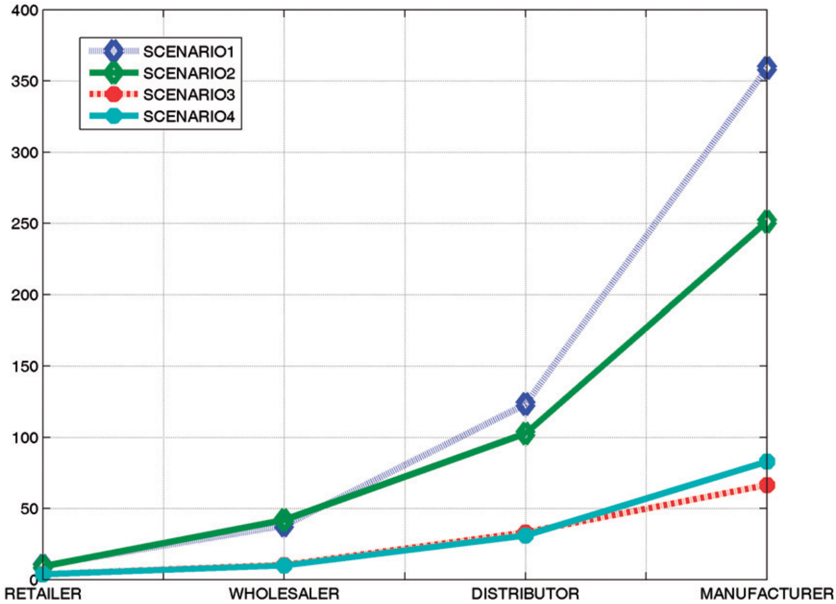

Figure 5. Increasing of mean bullwhip values going upstream in the supply chain in the case of the multi-stage model.

This definition cannot be applied to the retailer level since the final consumer does not have any inventory.

In Figure 6 the inventory oscillations mean values are represented for wholesaler, distributor and factory for every demand scenario. Again, as in the case based on orders, the factory is the level which amplifies least the oscillation in the inventories and this phenomenon increases going downstream in the supply chain.

According to the multi-stage model, the inventory oscillations suffered by each supply chain level (IOS_i), is calculated as the ratio between the inventory variance of the level to the retailer inventory variance (i.e. to the inventory variance of the nearest level to the final consumer):

$$\frac{IOS_i = \text{Var(inventory of level `}i\text{')}}{\text{Var(inventory of retailer)}} \tag{12}$$

In all considered scenarios the distributor is the level that mainly suffers the inventory oscillations as Figure 7 depicts.

The advantage of the factory in comparison to the distributor is due perhaps to the hypothesis of there being no production limit. This hypothesis does not affect the orders that the factory sends to production, but allows the factory to damp the oscillations induced by the distributor orders before they reach the inventory manufacturer level. This means that the factory without capacity constraints behaves like a filter of the inventory variance amplifications. Then the factory, which is the last level of the supply chain, suffers a greater variability in demand than the other areas but not in the inventories.

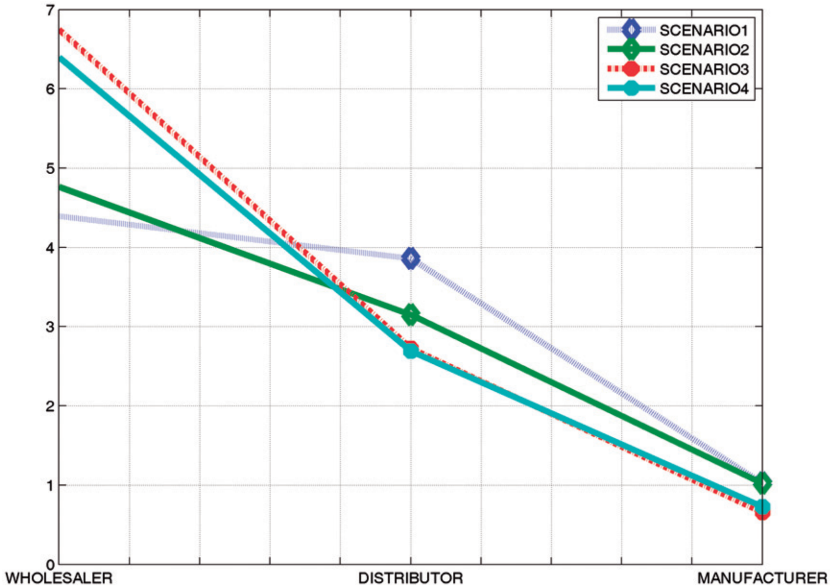

Figure 6. Average values of inventory oscillations with respect to supply chain sector, single-stage model.

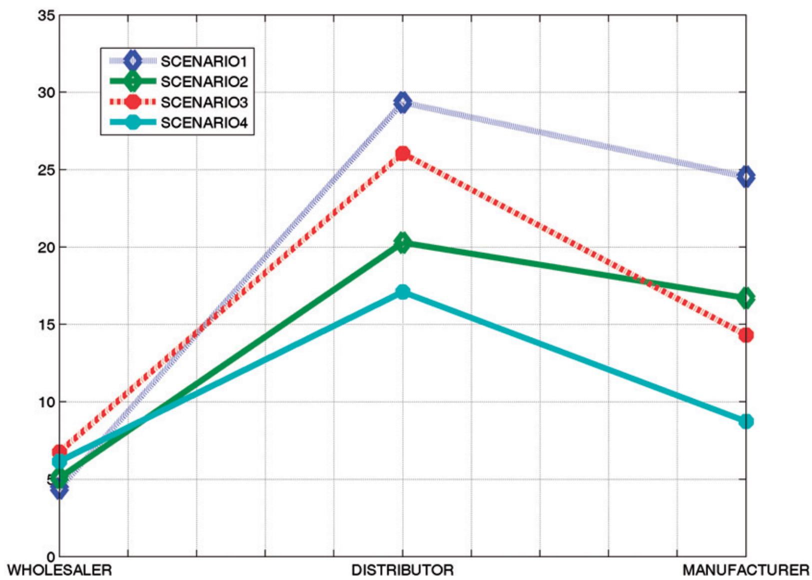

Figure 7. Average values of inventory oscillations suffered by the actors in a multi-stage model.

5. Generated costs and effective inventory maximum oscillation analysis

We are now interested in calculating the costs supported from each level to manage its inventory in each considered scenario. The aim is to see if there is a relationship between the bullwhip effect and the costs the actors must face. As we know, the objective of the

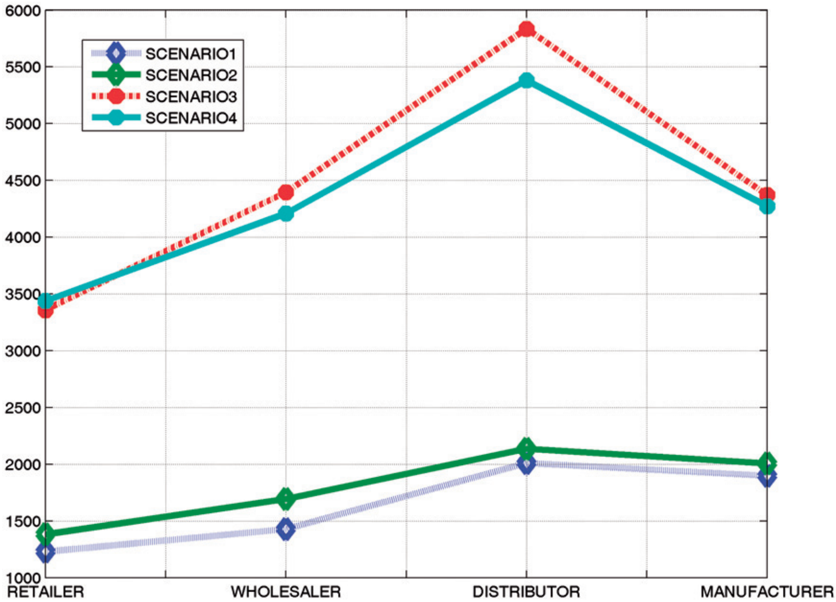

Figure 8. Average management costs of the inventory for each level.

single actor is to manage his inventory so as to minimise a cost function during the considered time horizon (60 weeks). The total cost for each level at the end of week '*t*' is given by the sum of the inventory and backlog costs:

$$\text{Cost}(t) = \text{Inventory}(t) * 0.50 + \text{Back log}(t) * 2 \tag{13}$$

Analysing the costs for the stock associated with each level of the supply chain, when the parameters α_S and β change simultaneously between 0 and 1, we obtain the mean values represented in Figure 8.

The distributor is always the level that faces more elevated costs for managing inventory. The reason for this is probably the higher variability in the inventory suffered at the distributor level than can be detected using the new measure of inventory oscillations, i.e. Equation (12) (see Figure 7). Moreover, we can observe that this behaviour is independent of the considered demand scenario and it seems more related to the unlimited factory production assumption that allows the replenishment of inventory when requested. It is not strange that the suffered inventory oscillations measure (Equation (12)) better corresponds to the costs of each level in comparison with the suffered bullwhip measure (Equation (10)), since inventories are considered in the cost calculation.

In order to complete our study, we also calculate the maximum oscillations in the stock level during the 60 weeks. When the inventory is equal to zero, backlogs are created: therefore, the single consideration of the inventory does not allow it to quantify the eventual gravity of the situation in a clear and complete way. In order to consider at the same time inventory and backlogs, the measure introduced by Caloiero *et al.* (2008) is used in this analysis. In particular, the effective inventory $(INV_t - BL_t)$ maximum oscillations have been considered as

$$\max_t(INV_t - BL_t) - \min_t(INV_t - BL_t) \tag{14}$$

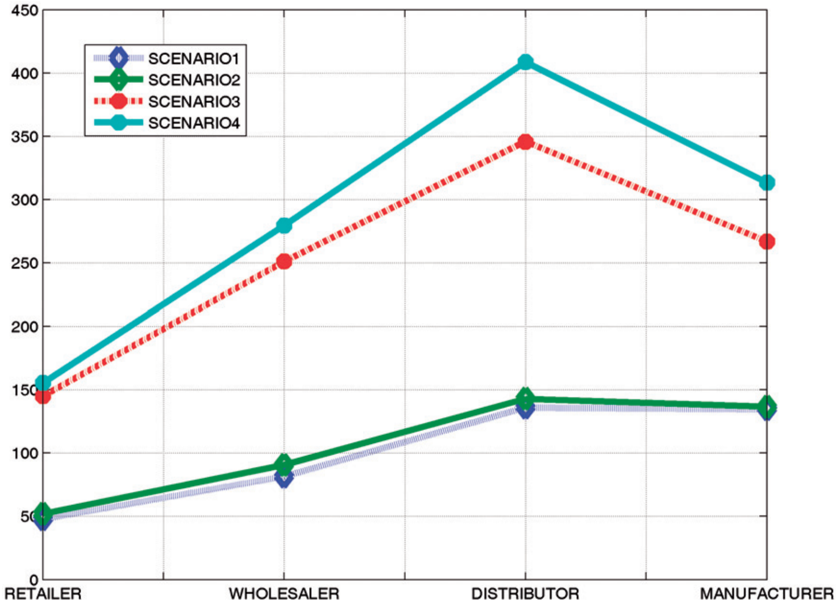

Figure 9. Average inventory maximum oscillations with respect to supply chain sector.

Average values of maximum oscillations are represented in Figure 9. It is possible to observe that they give the same information as the costs presented in Figure 8.

Once again, as we can see from Figure 9, the distributor is the level that faces the widest inventory oscillations. Since the factory is not characterised by production capacity constraints, as highlighted in Section 4, the manufacturer can maintain, better than the distributor, his inventory level near the desired one. As a consequence, the difference between his maximum and minimum effective inventory levels is smaller than the one characterising the distributor, and this explains why the maximum oscillation drops off at the manufacturer stage. The wider the oscillations, the greater the associated costs for the stock.

Using maximum oscillations and considering backlog, we can improve the description of the costs depending on the demand scenario and we can see that the distributor is always the actor with the higher costs (Figure 8). Moreover, using the extended definition of the bullwhip (Equations (11) and (12)), we can observe that the distributor has the widest oscillations in the inventory (Figure 7), but that the distributor is not the main actor in their generation (Figure 6).

6. Summary

In this work we have demonstrated that, with reference to the phenomenon known as the bullwhip effect, supply chains are unfair systems. In fact, by means of a bullwhip analysis performed on the beer game supply chain according to a single-stage and a multi-stage model, we have shown that the levels that are more responsible for the bullwhip generation are those that suffer less from it. In particular we have found that the bullwhip generated and suffered by each level exponentially decreases (see Figure 4) and increases (see Figure 5), respectively, going upstream in the supply chain. As a consequence the factory is the level most hit by the bullwhip effect and it is the one least responsible for its generation. Here, it is worth noting that this first outcome of our work, i.e. the proof of an inverse relation between generated and suffered bullwhip, confirms one of the most important behavioural hurdles characterising supply chains: the inability of each stage to learn from its actions, since the most relevant impact (of the actions) occur elsewhere in the chain (Chopra and Meindl 2001).

Moreover, in this paper we also focus on a new definition of inventory oscillations that is an extension of the bullwhip definition. As it is well known, the bullwhip based on orders is not always a good measure of the performances of the different supply chain levels. Since, in the beer game supply chain, the objective of each actor is to manage his inventory so as to minimise a cost function, we have tried to define a new indicator capable of providing some insight into how the different supply chain actors perform in terms of costs. In particular, while the cost function is given by the sum of inventory and backlog costs, we have defined an inventory oscillation measure as the bullwhip but for inventories instead of orders. At first glance, we have not considered backlogs into the proposed inventory oscillations quantification. By means of such a new measure, we have studied again the relation between the generated and the suffered model. We have found that the inventory oscillations are suffered more at the distributor level, which, again, is not the level that has the major responsibility for their generation. The effectiveness of the new measure has been demonstrated by the fact that the cost function along the supply chain follows a similar pattern as the suffered inventory oscillations defined in Equation (12) (see Figures 7 and 8) and the distributor is who spends more to manage his inventory.

Finally, to take into account the backlog costs, we have applied to the beer game supply chain presented in Section 2 the bullwhip analysis exploiting the measure introduced by Caloiero *et al.* (2008) in which the effective inventory maximum oscillations are considered. We have found that this measure has a complete correspondence with the cost trend along the supply chain, and, at least in the cases considered that measure can differentiate even the demand pattern. Notwithstanding the higher accuracy of the measure of Caloiero *et al.* (2008) in detecting the trend of the costs along the supply chain, the measure we have proposed seems more suitable to a real industrial context where the information on backlogs is often not available.

This paper would be incomplete if we did not mention the limitations of our model and analysis. The main drawback of the latter is given by the fact that we have not considered the impact of information quality, i.e. of centralised demand information on the generated and suffered bullwhip. However, extending our results to the centralised demand information case is straightforward and we have already planned this as the next research step. With reference to our model, the main limitations deal with the simplifying hypotheses of the beer game. Among them, it is worth mentioning the use of a simple exponential smoothing forecasting model and the manufacturer's infinite production capacity. In particular, in the cases of cyclic demand, a different forecasting model, the Winters model for instance, could be more suitable. Nevertheless, the results obtained in the four demand scenarios, even if different in values, are characterised by the same trend along the supply chain (see Figures 4 to 9). As a consequence, since for a step demand the simple exponential smoothing forecasting model is appropriate, we could conclude that by using a better fitting forecasting model for the cyclic demand scenario the levels more responsible for the bullwhip generation are those that suffer less from it. Finally, the unlimited production assumption for the manufacturer could be responsible for the trend of the inventory oscillations measure, which increases from the retailer to the distributor and then decreases at the manufacturer. As a matter of fact, an infinite production capacity could allow the replenishment of the manufacturer inventory when requested, and, consequently, the smoothing of the inventory variance increase as well as the increase of the maxima inventory oscillations. To confirm or disconfirm such hypothesis, a study of the generated and suffered inventory oscillation measure in the case of production capacity constraints at the manufacturer has been already planned.

Acknowledgements

The authors acknowledge Robin Donald for the review of English language; FS and TR acknowledge Fondazione Cariplo for financial support.

References

Baganha, M.P. and Cohen, M.A., 1998. The stabilising effect of inventory in supply chains. *Operations Research*, 46 (3), S72–S83.

Caloiero, G., Strozzi, F., and Zaldivar Comenges, J.M., 2008. A supply chain as a series of filter or amplifier of bullwhip effect. *International Journal of Production Economics*, 11 (4), 631–645.

Chatfield, D.C., *et al.*, 2004. The Bullwhip effect – Impact of stochastic lead time, information quality, and information sharing: a simulation study. *Production and Operation Management*, 13 (4), 340–353.

Chen, Y.F. and Disney, S.M., 2007. The myopic Order-Up-To policy with a proportional feedback controller. *International Journal of Production Research*, 45 (2), 351–368.

Chen, Y.F., *et al.*, 2000. Quantifying the bullwhip effect in a simple supply chain: the impact of forecasting, lead times, and information. *Management Science*, 46 (3), 436–443.

Chopra, S. and Meindl, P., 2001. *Supply chain management. Strategy, planning and operation*. Upper Saddle River, NJ: Prentice Hall.

Dejonckheere, J., *et al.*, 2004. The impact of information enrichment on the Bullwhip effect in supply chains: a control engineering perspective. *European Journal of Operational Research*, 153 (3), 727–750.

De La Fuente, D. and Lozano, J., 2007. Application of distributed intelligence to reduce the bullwhip effect. *International Journal of Production Research*, 45 (8), 1815–1833.

Disney, S.M. and Towill, D.R., 2003. On the bullwhip and inventory variance produced by an ordering policy. *OMEGA: The International Journal of Management Science*, 31 (3), 157–167.

Forrester, J.W., 1961. *Industrial dynamics*. Cambridge, Boston, MA: MIT Press.

Geary, S., Disney, S.M., and Towill, D.R., 2006. On bullwhip in supply chains – historical review, present practice and expected future impact. *International Journal of Production Economics*, 101 (1), 2–18.

Kahn, J., 1987. Inventories and the volatility of production. *American Economic Review*, 77 (4), 667–679.

Lee, H.L., Padmanabhan, V., and Whang, S., 1997. Information distortion in a supply chain: the bullwhip effect. *Management Science*, 43 (4), 546–558.

Lee, H.L., So, K.C., and Tang, C.S., 2000. The value of information sharing in a two-level supply chain. *Management Science*, 46 (5), 626–643.

Metters, R., 1997. Quantifying the bullwhip effect in supply chains. *Journal of Operations Management*, 15 (2), 89–100.

Mosekilde, E. and Larsen, E.R., 1988. Deterministic chaos in the beer production-distribution model. *System Dynamics Review*, 4 (1–2), 131–147.

O'Donnell, T., *et al.*, 2006. Minimising the bullwhip effect in a supply chain using genetic algorithms. *International Journal of Production Research*, 44 (8), 1523–1543.

Sarimveis, H., *et al.*, 2008. Dynamic modeling and control of supply chain systems: a review. *Computers & Operations Research*, 35 (11), 3530–3561.

Sterman, J.D., 1989. Modelling managerial behaviour: misperceptions of feedback in a dynamic decision-making experiment. *Management Science*, 35 (3), 321–339.

Strozzi, F., Zaldivar Comenges, J.M., and Noè, C., 2008. Stability control in a supply chain: total costs and bullwhip effect reduction. *The Open Operational Research Journal*, 2 (7), 51–59.

Thomsen, J.S., Mosekilde, E., and Sterman, J.D., 1992. Hyperchaotic phenomena in dynamic decision making. *Journal of Systems Analysis and Modelling Simulation (SAMS)*, 9, 137–156.

Tzafestas, S., Kapsiotis, G., and Kyriannakis, E., 1997. Model-based predictive control for generalised production planning problems. *Computers in Industry*, 34 (22), 201–210.

Xiong, G. and Helo, P., 2006. An application of cost-effective fuzzy inventory controller to counteract demand fluctuation caused by bullwhip effect. *International Journal of Production Research*, 44 (24), 5261–5277.

Appendix

In Table 1 the list of all the symbols and acronyms used is provided in alphabetical order.

Table 1. List of symbols and acronyms.

Symbol or acronym	Meaning
α_S	Stock adjustment rate ($0 \leq \alpha_S \leq 1$).
α_{SL}	Stock in transit adjustment rate ($0 \leq \alpha_{SL} \leq 1$).
AS_t	Stock adjustment at period t. $AS_t = \alpha_S * (DINV - INV_t + BL_t)$.
ASL_t	Stock in transit adjustment at period t. $ASL_t = \alpha_{SL} * (DSL - SL_t)$.
β	Ratio between the stock in transit adjustment rate and the stock adjustment rate.
BL_t	Backlog level experienced by the actor at time t.
BOG_i	Bullwhip generated by the level i. BOG_i = var (order placed by level 'i')/var (order placed by level '$i-1$').
BOS_i	Bullwhip suffered by the level i. BOS_i = var (order placed by level 'i')/var (COR).
C_{BL}	Unitary backlog cost.
C_{INV}	Unitary inventory cost.
COR	Customer order rate.
$COST$	Costs the actor must face. $COST = \Sigma_t INV_t * C_{INV} + \Sigma_t BL_t * C_{BL}$.
$DINV$	Desired inventory level of the actor.
DSL	Desired stock in transit of the actor.
ED_t	Expected demand at period t.
INV_t	Inventory level experienced by the actor at time t.
IO_t	Incoming orders at time t.
IOG_i	Inventory oscillations generated by the level i. IOG_i = var (inventory of level 'i')/var (inventory of level '$i-1$').
IOS_i	Inventory oscillations generated by the level i. IOS_i = var (inventory of level 'i')/var (inventory of retailer).
O_t	Order placed by the actor at period t.
Q	Order-up-to point quantity. $Q = DINV + \beta * DSL$.
SL_t	Stock in transit directed toward the actor at time t.
θ	Parameter of the exponential smoothing forecasting technique ($0 \leq \theta \leq 1$). It represents he weight given to the incoming orders with respect to the expected demand.